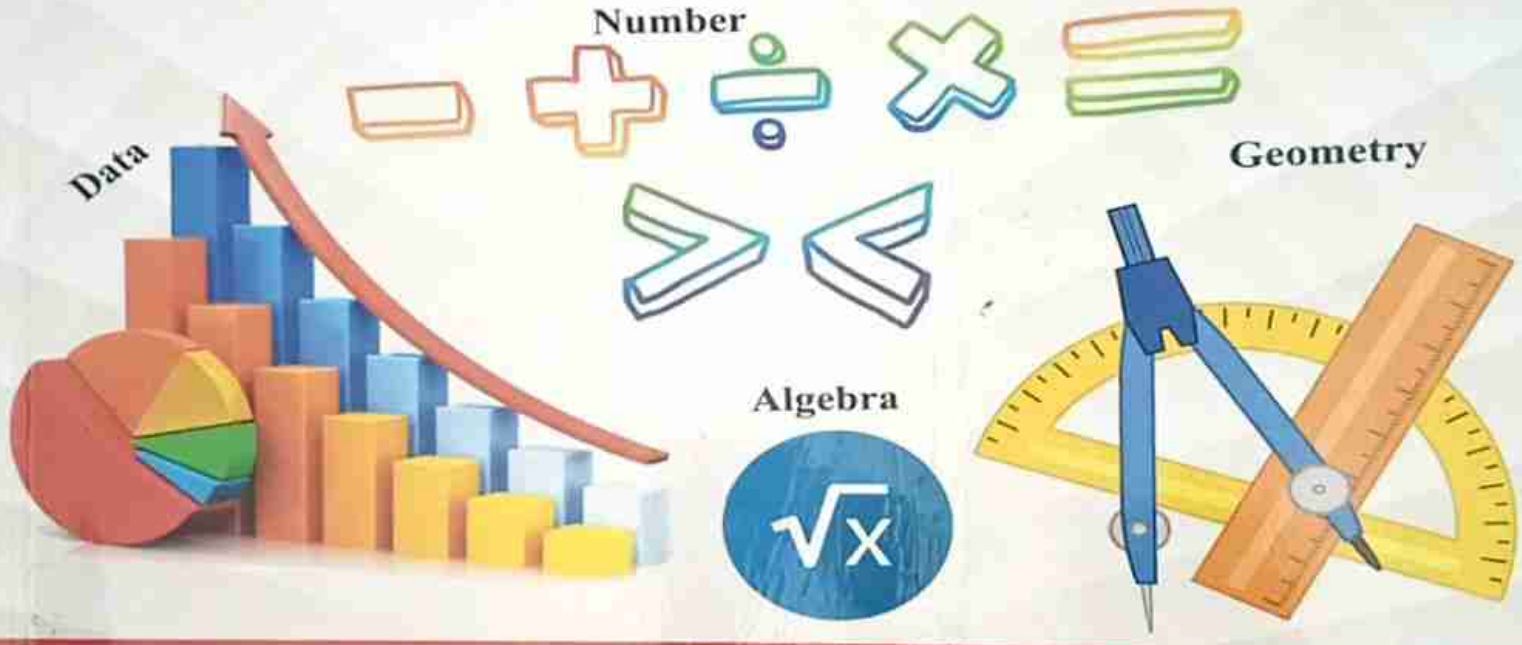
FOR GHANA SCHOOLS

BEST BRAIN

MATHEMATICS

FOR BASICS 7, 8 & 9



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Henric Atta Baah-Yeboah

BS 8

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BASIC 8

STRAND 1: NUMBER

SUB-STRANDS

- Number and numerations systems
- Number operations
- Fractions, decimals and percentages
- Ratio and proportion

STRAND 1.1 NUMBER AND NUMERATION SYSTEM

B8.1.1.1.1 Apply place value to read and write number quantities over 1,000,000,000

Developing the concept of reading and writing numbers quantities

In Basic 7, we learnt about modelling number quantities. We also learnt about number names and reading out given number quantities. As number quantities get larger, reading them out or writing them in words can be challenging sometimes. This is because we always need to differentiate between the "face value" of the digits and their "place value". Using our knowledge on place value will help us to be able to read and write number quantities.

Let us remind ourselves of the place value chart.

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
7	7	7	7	7	7	7	7	7	7

Looking at the chart, the number 7 repeats for all the places but its value is not the same for each of them. For example, the 7 under the tens place has a value of 70, but the 7 under the millions Place has the value of 7,000,000.

Let us take a look at this reorganised chart.

Place Value Chart

Millions			Thousand			Ones		
Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
7	7	7	7	7	7	7	7	7

We have now grouped the places in the chart. We will learn to read and write quantities using these groupings. These groups are important because they help us to know where to place "commas" and how to partition the name correctly.

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The "ones" group has hundreds, tens and ones.

Example 417

Ones		
Hundreds	Tens	Ones
8	5	9

The number name for this number 859 is "eight hundred and fifty nine".

Take note that we separate the "hundred" from the "tens and ones" with "and".

Example 418

Ones		
Hundreds	Tens	Ones
2	4	7

The number name for this number 247 is "two hundred and forty seven".

The "thousands" group has hundred thousands, ten thousands and thousands.

Example 419

Thousands		
Hundred Thousands	Ten Thousands	Thousands
8	5	9

The number name for this number 859,000 is "eight hundred and fifty nine thousand". Note that the three zeros "000" are representing the places for hundreds, tens and ones.

Once again, we separate the "hundred thousand" from the "tens and ones" with "and" when reading or writing the name.

Example 420

Thousands		
Hundred Thousands	Ten Thousands	Thousands
2	4	7

The number name for this number in the chart 247,000 is "Two hundred and forty seven thousand".

The "millions" group has hundred million, ten million and million.

Example 421

Hundred Millions	Ten Millions	Millions
7	1	7

The number name for this number 717,000,000 is "seven hundred and seventeen million". Note that the six zeros "000,000" are representing the places for hundred thousand, ten thousand, thousand, hundred, tens and ones.

Now let's name quantities without the place value chart. We will use commas to guide us instead.

Example 422

Write the number name for 312,580,747

The number 312,580,747 has three groups of three: 312 (three hundred twelve), 580 (five hundred eighty), and 747 (seven hundred forty-seven).

The first comma signifies million (three hundred twelve million) and the second comma signifies thousand (five hundred eighty thousand). The last group is simply seven hundred forty-seven. Combine them and you get. Three hundred and twelve million, five hundred and eighty thousand, seven hundred and forty-seven.

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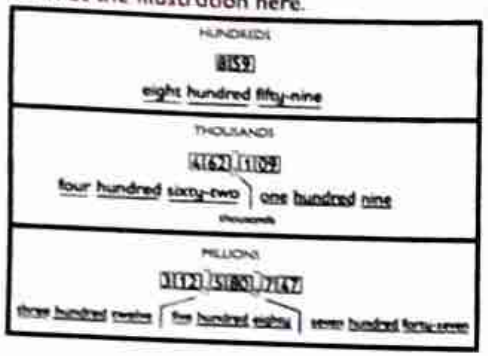
Note: when writing the name, use commas to separate the groups (i.e. Million, thousands and ones), then use 'and' to separate the 'hundred' from the others in each group.

Look at the illustration here.



311 = three hundred and eleven
2,311 = two thousand, three hundred and eleven
52,311 = fifty-two thousand, three hundred and eleven
952,311 = nine hundred and three hundred and eleven thousand
8,952,311 = eight million, two thousand, three hundred and eleven
68,952,311 = sixty-eight million, nine hundred and fifty-two thousand, three hundred and eleven
468,952,311 = four hundred and sixty-eight million, nine hundred and fifty-two thousand, three hundred and eleven.

Example 423

We may also use another strategy where we breakdown the number into smaller numbers. Write and read out the number name for the quantity 468,952,311.

In Basic 6, we used our knowledge on place value to read and write numbers up to 1,000,000,000. In this section, we shall read and write number quantities over 1,000,000,000,000 using our knowledge on place value. Look at the place value chart below.

Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
4	2	8	4	6	1	9	3	5	2

The number quantity written in the place value chart is 4,284,619,352
This number has a number name: Four billion, two hundred and eighty-four million, six hundred and nineteen thousand, three hundred and fifty-two.

Example 424

Write the place and value of 7 in the number 8,314,972,651

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Solution

Using a place value chart, we have

Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
8	3	1	4	9	7	2	6	5	1

The place of 7 is ten-thousands and the value of 7 is 7 ten-thousand (7 x 10,000) or 70,000

Example 425

Write the place and value of 3 and 4 in the number 35,896,654,076

Solution

Using a place value chart, we have

Ten - Billions	Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
3	5	8	9	6	6	5	4	0	7	6

The place of 3 is ten-billions and the value of 3 is 3 ten-billion or 30,000,000,000
The place of 4 is thousands and the value of 4 is 4 thousand or 4,000

Example 426

Write the number name for the number 652,841,374

Solution

Using a place value chart, we have

Ten - Billions	Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
		6	5	2	8	4	1	3	7	4

The number name is Six hundred and fifty-two million, eight hundred and forty-one thousand, three hundred and seventy-four.

Example 427

Write the number name for the number 17,045,371,541

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Solution

Using a place value chart, we have

Ten - Billions	Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
1	7	0	4	5	3	7	1	5	4	1

The number name is: Seventeen billion, forty-five million, three hundred and seventy-one thousand, five hundred and forty-one.

Example 428

Write the numeral for the number name: Two hundred and sixty-eight thousand, seven hundred and thirty-two

Solution

The thousands part is two hundred and sixty-eight thousand: 268,000
The hundreds part is seven hundred: 700
The tens and ones part is thirty-two: 32
Therefore the numeral = 268,000 + 700 + 32
= 268,732

Example 429

Write the numeral for the number name: Nine hundred and fourteen billion, three hundred and five million, six hundred thousand, four hundred and one

Solution

The billions part is nine hundred and fourteen billion: 914,000,000,000
The millions part is three hundred and five million: 305,000,000
The thousands part is six hundred thousand: 600,000
The hundreds part is four hundred: 400
The tens and ones part is one: 1

The numeral = 914,000,000,000 + 305,000,000 + 600,000 + 400 + 1
= 914,305,600,401

EXERCISE 130 Date

1. Write the place and value of the underlined digits in each of the following.
   (a) 2<u>3</u>8,167 (b) 8,9<u>3</u>4,615
   (c) <u>5</u>2,342,714 (d) 1<u>4</u>2,854,387
2. Write the number name for each of the numerals.
   (a) 618,928 (b) 54,819,542
   (c) 674,819,081,621
3. Write the numeral for each of the number names.
   (a) Four hundred and six thousand, three hundred and thirteen
   (b) Nine hundred and ten million, seven hundred and forty-five thousand and two
   (c) Eight hundred and five billion, sixty-one million, four hundred and twelve thousand nine hundred and three
   (d) Four hundred and six million, seven hundred and thirteen.

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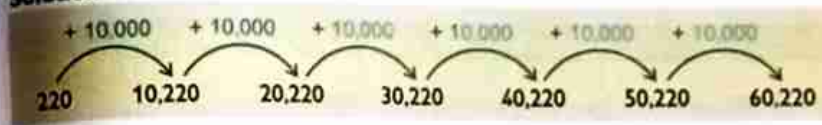
B8.1.1.1.2 Skip count forwards and backwards in 10,000, 100,000, 500,000, etc.

Skip counting forward or backwards is a familiar concept right from Basic 1. In this section, we shall use the same concept to skip count numbers in 10,000, 100,000, 500,000, etc. Remember that to skip count forward we add and to skip count backwards, we subtract.

Example 430

Count forward in 10,000 up to the sixth number starting from 220.

Solution



Example 431

Consider the numbers: 10,000, 110,000, 210,000, ____, ____, ____
Find the number in which they are skip counted on and complete it.

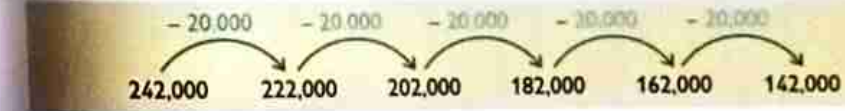
Solution

We can observe that the numbers are skip counted forward in 100,000
10,000, 110,000, 210,000, 310,000, 410,000, 510,000

Example 432

Count backwards in 20,000 up to the fifth number starting from 242,000

Solution



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Example 433

Consider the numbers: 98,000, 95,000, 92,000, ____, ____, ____
Find the number in which they are skip counted on and complete it.

Solution

We can observe that the numbers are skip counted backwards in 3,000
98,000, 95,000, 92,000, 89,000, 86,000, 83,000

EXERCISE 131 Date

1. Count forward in 30,000 up to the fifth number starting from 4,000.
2. Count backwards in 50,000 up to the fifth number starting from 861,000.
3. Count backwards in 10,000 up to the sixth number starting from 402,000.
4. Find the number in which the following are skip counted on and complete it.
   (a) 36,000, 37,000, 38,000, _____, _____, _____, _____, _____
   (b) 178,000, 173,000, 168,000, _____, _____, _____, _____, _____
   (c) 29,000, 529,000, 1,029,000, _____, _____, _____, _____, _____
   (d) 836,000, 816,000, 796,000, _____, _____, _____, _____, _____

B8.1.1.1.3 Compare and order whole numbers using >, < and =

In Basic 7, we compared two 8 or 9-digit numbers using more than or less than. In this section, we shall compare numbers using words such as less than or more than and <, > and = symbols.

Example 434

Compare 4,917,390 and 4,817,390 using 100,000 more than or 100,000 less than.

Solution

Comparing the numbers using place value chart.

Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
4	9	1	7	3	9	0
4	8	1	7	3	9	0

The numbers at the hundred-thousand place are 9 and 8 but 9 > 8. The difference between 9 and 8 is 1.
Therefore, 4,917,390 is 100,000 more than 4,817,390.

Example 435

Write the number name for the number 652,841,374

Solution

Compare the numbers using place value chart

Ten - Billions	Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
1	7	9	8	3	2	1	4	4	3	8
1	7	9	8	3	7	1	4	4	3	8

The numbers at the hundred-thousand place are 2 and 7 but 2 < 7. The difference between 7 and 2 is 5.
Therefore, 17,983,214,438 is 500,000 less than 17,983,714,438

Example 436

Compare 15,876,451,716 and 15,081,561,399 using the symbol ">", "<" or "=".

Solution

Compare the numbers using place value chart.

Ten - Billions	Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
1	5	8	7	6	4	5	1	7	1	6
1	5	0	8	1	5	6	1	3	9	9

Comparing the numbers at the hundred-millions place are 8 and 0 but 8 > 0

Therefore, 15,876,451,716 > 15,081,561,399

Example 437

Compare 602,816,471 and 602,816,471 using the symbol ">","<" or "=".

Solution

Compare the numbers using place value chart.

Ten - Billions	Billions	Hundred - Millions	Ten - Millions	Millions	Hundred - thousands	Ten - thousands	Thousands	Hundreds	Tens	Ones
1	5	8	7	6	4	5	1	7	1	6
1	5	0	8	1	5	6	1	3	9	9

The numbers at the various places in the place value chart are the same.

Therefore 602,816,471 = 602,816,471

EXERCISE 132 Date

1. Compare 34,189,617 and 34,179,617 using 10,000 more than or 10,000 less than.
2. Compare 671,042,483 and 671,092,483 using 50,000 more than or 50,000 less than.
3. Compare 8,134,862,451 and 8,134,662,451 using 200,000 more than or 200,000 less than.
4. Compare 60,514,300 and 60,524,300 using the symbol ">","<" or "=".
5. Compare 745,098,423 and 745,091,876 using the symbol ">","<" or "=".
6. Compare 412,052,312,098 and 412,019,986,577 using the symbol ">", "<" or "=".

Real world problems on comparing number quantities

Now that we can compare numbers that are 100,000 more or less than a quantity using the place value chart, we will explore some real world problems that involve comparing large quantities.

Example 438

Three business partners, Akwesi, Kawiah and Akosua were paid GH¢ 23,451,702, GH¢ 23,351,702 and GH¢ 23,551,702 respectively for a road construction project.

a) **Determine how much more or less Akwesi had than Kawiah.**

b) **Compare and arrange the amount each received in ascending order of magnitude using the appropriate symbol (<,>,or =).**

Solution

a) Akwasi – GH¢23,451,702
Kawiah – GH¢23,351,702
Comparing intervals, Akwasi had 100,000 more than Kawiah.

b) Using symbols to order the amount each received, we have
GH¢23,351,702< 23,451,702 < 23,551,702

Example 439

Dankwa is due to receive an increment of GH¢500,000 annually for his new contract at his football club.

a) **If his wage is GH¢1,355,000, how much will he receive in his new contract?**

b) **If the football club eventually pays Dankwa an amount of GH¢1,755,000, how much more or less have they paid as compared to the agreed amount in the contract?**

Solution

a) He will receive GH¢1,355,00 + GH¢500,000 = GH¢1,855,000

b) Comparing 1,755,000 and 1,855,000 we can see that the new payment of 1,755,000 is 100,000 less than the old payment of GH¢1,855,000

Example 440

I want to buy a house with least price. Which one should I buy?

A. GH¢145,634,800;
B. GH¢145,635,000;
C. GH¢145,535,000

Solution

Using place value chart to order the numbers, we can see that the least price is GH¢145,535,000.

EXERCISE 133 Date

1. Given below is the population of 3 virtual cities in a computer game. Use the data in the table to answer the questions that follow

City	Population
New York	50,607,310
Virginia	50,077,319
Florida	50,617,310

Which city has the largest population?
How much more or less is the population of NewYork than Florida?
Compare and arrange their population in descending order of magnitude.

2. Given below is the distance of 3 towns from Salaga.
   a) The closest town is

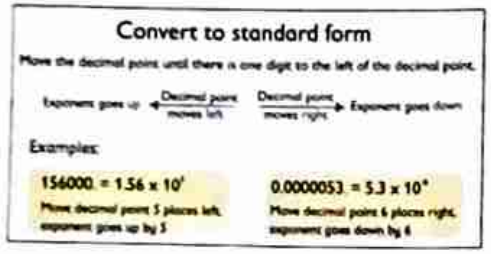
A. Town X - 188,528,500 m	B. Town Y - 186,528,500 m	C. Town Z - 187,528,500 m

   b) How much more or less is Town X from TownY?

B8.1 1.1.4 Express integers into standard form

Standard form is also known as standard index form or scientific notation. It involves writing large numbers or very small numbers in terms of powers of 10.

The following diagram shows how to convert numbers to standard form.


Convert to standard form
Move the decimal point until there is one digit to the left of the decimal point.
Exponent goes up
Decimal point moves left
Decimal point moves right
Exponent goes down
Examples:
156000. = 1.56 x 10^5
Move decimal point 5 places left, exponent goes up by 5
0.0000053 = 5.3 x 10^-6
Move decimal point 6 places right, exponent goes down by 6

The illustration should summarise how to convert a given quantity to standard form. If you are still not clear then let us go through the following activities for better clarification.

Numbers in standard form are written in this format:

$$a \times 10^{n}$$

Where 'a' is a number $1 \leq a < 10$ and n is an integer.

To do this we need to understand the place value of a number.

For example,

Let us look at the number 8290000 and write the digits in a place value table.

10^6	10^5	10^4	10^3	10^2	10^1	10^0
8	2	9	0	0	0	0

So, 8290000 written in standard form is 8.29×10^6

Standard form notation

Standard form notation is a representation of place value which compliments the decimal number system, as shown in the table below.

Words	Decimal form	Fraction form	Fraction form with powers of ten	Powers of ten	Standard form
Thousand	1000	1000	10^3	10^3	1×10^3
Hundred	100	100	10^2	10^2	1×10^2
Ten	10	10	10^1	10^1	1×10^1
Unit	1	1	10^0	10^0	1×10^0
Tenth	0.1	$\frac{1}{10}$	$\frac{1}{10^1}$	10^{-1}	1×10^{-1}
Hundredth	0.01	$\frac{1}{100}$	$\frac{1}{10^2}$	10^{-2}	1×10^{-2}
Thousandth	0.001	$\frac{1}{1000}$	$\frac{1}{10^3}$	10^{-3}	1×10^{-3}

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Any integer or terminating decimal can be written using the standard form notation $a \times 10^n$. The table below shows how the value of 'a' can remain the same while the power of ten 'n' changes the place value of those digits.

Decimal Form	Standard Form
7500	7.5×10^3
750	7.5×10^2
75	7.5×10^1
7.5	7.5×10^0
0.75	7.5×10^{-1}
0.075	7.5×10^{-2}
0.0075	7.5×10^{-3}

So, we realise from the table that the value of 'a' remained 7.5 for all but the power of 10 kept changing to correspond to the actual value in each case.

Using standard form notation enables us to write very large or very small numbers.

Example 441

a) $675000000000000000000000 = 6.75 \times 10^{23}$

b) $0.000000000000037 = 3.7 \times 10^{-14}$

Using standard form notation also enables us to compare the size of very large or very small numbers easily.

Example 442

Which is larger, 8560000000000 or 45320000000000?

Solution

At a glance it is difficult to tell which is larger, but written in standard form you can compare these numbers very quickly as shown.

Which is larger, 8.56×10^{12} or 4.532×10^{13}?

Instantly, we can see that 4.532×10^{13} is the larger number as it has the higher power of ten.

Summary of steps

In order to write numbers in standard form,

1. identify the non-zero digits and write these as a decimal number which is greater than or equal to 1 but less than 10.
2. in order to maintain the place value of the number, this decimal number needs to be multiplied by a power of ten.
3. write the power of ten as an exponent.
4. write your number in standard form.

In order to convert from standard form to ordinary numbers,

1. convert the power of ten to an ordinary number.
2. multiply the decimal number by this power of ten.
3. write your number as an ordinary number.

Standard form calculation

When numbers are written in standard form, it can make some calculations neater and quicker to compute. This is particularly the case for calculations involving multiplication, division and powers. However, this depends on an understanding of the rules of indices. Therefore, after you have studied the concept of indices and its laws, come back and go through these activities.

Multiplication and division of standard form

In order to multiply and divide in standard form,

1. multiply or divide the non-zero numbers.
2. multiply or divide the powers of ten by adding or subtracting the indices.
3. write the solution in standard form, checking that the first part of the number is $1 \leq x < 10$.

BEST BRAIN

Example 443

Work out $(4 \times 10^3) \times (2 \times 10^8)$

Solution

$4 \times 2 = 8$

$10^3 \times 10^8 = 10^{11}$

8×10^{11} is in standard form.

Example 444

Calculate $(5 \times 10^3) \times (7 \times 10^8)$

Solution

$5 \times 7 = 35$

$10^4 \times 10^8 = 10^{12}$

35×10^{12} is not in standard form.

To convert 35 to a decimal between 1 and 10 you need to divide it by 10. To maintain the value of the number, you need to multiply the power of ten by 10, which adds one to the exponent.

The answer is 3.5×10^{13}.

Example 445

Work out $(2 \times 10^3) + (4 \times 10^8)$

Solution

1. $2 \div 4 = 0.5$
2. $10^3 \div 10^8 = 10^{-5}$
3. 0.5×10^{-5} is not in standard form

0.5 is less than 1 so multiply 0.5 by 10 which gives the result 5. To compensate, you need to divide the power of ten by 10 which has the effect of subtracting one from the power, which gives the result 10^{-6}.

So the answer is 5×10^{-6}.

Example 446

Calculate $(8 \times 10^3) \div (2 \times 10^5)$.

Solution

$8 \div 2 = 4$

$10^3 \div 10^5 = 10^{-2}$

4×10^{-2}

This is in standard form as 4 is greater than 1 but less than 10.

Addition and subtraction in standard form

In order to add and subtract numbers in standard form,

1. convert the number with the lower power of ten so that both numbers have the same power of ten.
2. add the non-zero integers.
3. check if your answer is in standard form

Example 447

Workout $(3 \times 10^3) + (6 \times 10^4)$

Solution

$6 \times 10^4 = 0.6 \times 10^5$

$3 + 0.6 = 3.6$

3.6×10^5 is in standard form.

Example 448

Workout $(6 \times 10^3) + (2 \times 10^5)$

Solution

2×10^5

You need to increase the power. To do this multiply it by 10 to add one to the power. But to maintain the value of the number you need to divide the non-zero number by 10.

2×10^5

$\div 10 \quad \times 10$

0.2×10^6

6×10^4 and 0.2×10^6

You now add the two non-zero numbers

$6 + 0.2 = 6.2$

So your answer in standard form is 6.2×10^4.

BEST BRAIN

Example 449

Workout $(9 \times 10^4) - (3 \times 10^3)$

Solution

$3 \times 10^3 = 0.3 \times 10^4$

$9 - 0.3 = 8.7$

8.7×10^4 is in standard form.

Example 450

Workout $(5 \times 10^4) - (4 \times 10^3)$.

Solution

To do this identify the number with the lowest exponent.

4×10^3

You need to increase the power. To do this multiply it by 10 to add one to the power. But to maintain the value of the number you need to divide the non-zero number by 10.

4×10^3

$\div 10 \quad \times 10$

0.4×10^4

5×10^4 and 0.4×10^4

You now subtract the two non-zero numbers

$5 - 0.4 = 4.6$

So your answer in standard form is 4.6×10^4.

EXERCISE 134 Date

Try your hands on these real-life examples

1. The distance from Earth to Mars is 288,000,000 km, to three significant figures. Write this number in standard form.
2. The speed of light is 3×10^8 m/s, rounded to one significant figure. Write this as an ordinary number.
3. The mass of Earth is 5.97×10^{24} kg. Work out the mass of the Earth in grams. Write your answer in standard form.
4. Put these numbers in order. Start with the smallest number.

   62×10^{-3}, 0.0068, 6.9×10^{-1}, 0.0607

   (Hint: Convert all numbers to standard form first)

BEST BRAIN

Let us see how the following numbers can be written in standard form

$1 = 1 \times 10^0$

$10 = 1 \times 10^1$

$100 = 1 \times 10 \times 10 = 1 \times 10^2$

$1{,}000 = 1 \times 10 \times 10 \times 10 = 1 \times 10^3$

Let us use the ideas used in the explanation on how to write numbers in standard form to solve more problems.

Example 451

Write 4,520 in standard form.

Solution

$4520 = 4520.0$

$= 4\overset{3}{\overset{\wedge}{5}}\overset{2}{\overset{\wedge}{2}}\overset{1}{\overset{\wedge}{0}}.0$

$= 4.520 \times 10^3$

Example 452

Write 0.0078 in standard form.

Solution

$0.00078 = 0.\overset{\wedge}{0}\overset{\wedge}{0}\overset{\wedge}{0}\overset{\wedge}{7}\overset{\wedge}{8}$

$= 7.8 \times 10^{-4}$

Example 453

Express 5 days in minutes, leave the answer in standard form.

Solution

5 days $= 5 \times 24$ hours

$= 5 \times 24 \times 60$ minutes

$= 7{,}200$ minutes

Therefore $7{,}200 = 7.2 \times 10^3$ minutes

Example 454

Express 107 m 86 cm in centimetres, leave the answer in standard form.

Solution

107 m 86 cm $= 107 \times 100$ cm $+ 86$ cm

$= 10700 + 86$

$= 10{,}786$ cm

Therefore $10{,}786 = 1.0786 \times 10^4$ cm

Example 455

Write 0.137×0.08, leave the answer in standard form.

Solution

$$0.137 \times 0.08 = \frac{137}{1000} \times \frac{8}{100}$$

$$= \frac{137 \times 8}{1000 \times 100}$$

$$= \frac{1096}{100000}$$

$$= 0.01096$$

$$= 1.096 \times 10^{-2}$$

EXERCISE 135 Date.....

1. Write 300,000 in standard form.
2. Write 82,189 in standard form.
3. Write 0.000008 in standard form.
4. Write 0.0387 in standard form.
5. Write 0.0002971 in standard form.
6. Express 28 days in hours, leave the answer in standard form.
7. Write 0.561×0.025, leave the answer in standard form.
8. Express 93 kg 526 g in grams, leave the answer in standard form.

B8.1.1.1.5A Express integers to a given number of decimal places

In Basic 7, we learnt how to write a given decimal numeral to a given decimal places. Remember we said that the numeral 234 can be written to 1 decimal place as 234.0. In this section, we shall use the same concept to write integers to a given decimal places.

Example 456

Express 45.938 correct to 2 decimal places.

Solution

Two decimal places of the numeral 45.938 is 45.93, but the number after 3 is 8 which is more than 5 so we shall add 1 to the 3.

Therefore 45.938 becomes 45.94

Example 457

Express 29.9995 correct to 3 decimal places.

Solution

Three decimal places of the numeral 29.9995 is 29.999, but the number after the last 9 is 5 so we shall add 1 to the last digit.

$$\begin{array}{r} 29.999 \\ + \quad 1 \\ \hline 30.000 \end{array}$$

Therefore 29.9995 becomes 30.000

Example 458

Find 0.28×0.7 correct the answer to 2 decimal places.

Solution

$$0.28 \times 0.7 = \frac{28}{100} \times \frac{7}{10}$$

$$= \frac{28 \cdot 7}{100 \cdot 10}$$

$$= \frac{196}{1{,}000}$$

$$= 0.196$$

Therefore $0.28 \times 0.7 = 0.20$ (to 2 decimal places)

Example 459

A man taped 208.672 litres of palm wine on Monday, 168.32 litres on each of the two days (Tuesday and Wednesday). Find

(a) the total litres of palm wine taped on the three days;

(b) write the answer in (a) to 1 decimal place.

Solution

(a) Total litres of palm wine taped on the three days

$= 208.672 + 168.32 + 168.32$

$= 208.672 + 336.64$

$= 545.312$ litres

(b) $545.312 = 545.3$ litres

EXERCISE 136 Date.....

1. Write 8,192 to 1 decimal place.
2. Express 0.8215 to 3 decimal places.
3. Write 78.976 to 2 decimal places.
4. Write 1835.996, correct to 2 decimal places.
5. Find 0.84×0.6, correct to 2 decimal places.
6. Find 0.562×8, correct to 1 decimal place.
7. A woman bought 876 litres of palm oil. She sold 56.28 litres of palm oil each hour for the first 7 hours. Find:
   (a) the number of litres of palm oil sold in the first 7 hours, correct to 1 decimal place.
   (b) the number of litres of palm oil left, correct to 2 decimal places.
8. A carpenter bought 321.876 cm of wood, he used 198.42 cm to prepare a table. Find the length of wood left, correct to 2 decimal places.

B8 1.1.1.5B Express integers to a given number of significant figures

In Basic 7, we learnt how to write a decimal numeral to a given significant figures. In this section, we shall use our knowledge from Basic 7 to write a given integer to specified significant figures.

Example 460

Express 32,852 to 3 significant figures.

Solution

Three significant figures is 328, and replace the remaining two numbers with zeros.
The remaining number is 52 which is more than 50, so we round it to 100.
We have 32,800 + 100 = 32,900
Therefore 32,852 becomes 32,900

Example 461

Express 8,942,472 to 4 significant figures.

Solution

Four significant figures is 8942, and replace the remaining three numbers with zeros.
The remaining number is 472 which is less than 500, so we shall round it to zero.
We have 8,942,000 + 0 = 8,942,000
Therefore, 8,942,472 becomes 8,942,000

Example 462

A man bought 58 goats, if the mass of each goat is 47 kg, find the total mass of the goats correct to 3 significant figures.

Solution

Total mass of the goats = 58 × 47
= 2,726 kg
Three significant figures is 272, and replace the remaining number with zero.
The remaining number is 6 which is more than 5, so we shall round it to 10.
We have 2720 + 10 = 2730
Therefore, 2,726 kg becomes 2,730 kg.

EXERCISE 137 Date

1. Express 17,902 to 2 significant figures.
2. Express 42,714,552 to 4 significant figures.
3. Express 305,679,821 to 5 significant figures.
4. Express 69,800,164 to 1 significant figure.
5. A trader bought 27 bags of maize. If each bag of maize weighs 62 kg, find the total weight of all the 27 bags of maize, correct to 3 significant figures.
6. A man saves 20% of his income each month. If his income for a month is GH¢3,818.00. Find
   (a) the total amount saved in a year;
   (b) the total amount not saved, correct to 2 significant figures.

B8 1.1.1.6 Solve real life problems on place values

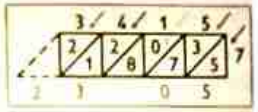
In this section, we shall use our knowledge on number name to find the total quantity of items in real life problem.

Example 463

A trader bought 3,415 tubers of yam at GH¢7.00 each. Find the total amount paid by the trader, write the answer in its number name.

Solution

Total amount paid
= 3415 × 7
= GH¢23,905.00



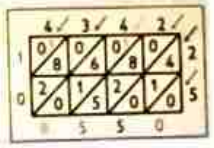
The amount paid is twenty-three thousand, nine hundred and five Ghana cedis.

Example 464

A farmer walked a distance of 4,342 m each day to his farm and back. Find the total distance walked after 25 days.

Solution

Total distance walked
= 4342 × 25
= 108,550 m



Example 465

A man is paid GH¢13,205.00 a month, and earns an additional allowance of GH¢3,805.00. Find his total salary for six months.

Solution

Total salary for 6 months
= 6 × 13205 + 6 × 3805
= 79230 + 22830
= GH¢102,060.00

EXERCISE 138 Date

1. A man was paid a retirement package of GH¢167,423.00. He used GH¢48,009.00 to complete his building. How much of his money will be left?
2. A business man bought 2,050,113 books to sell to the general public. He sold 13,542 books each week. Find the number of books left after 17 weeks.
3. A farmer harvested 52 bags of beans in a year. He kept 8 bags and sold the rest at GH¢425.00 each. Find the total amount of money he got.
4. A man earned GH¢628,555.00 in two years and the wife also earned GH¢701,238.00 in two years. Find how much more money did the woman earned than the man.
5. A woman sold 35 yards of kente cloth each day. Find the total number of yards of kente cloths sold in a year of 365 days.
6. A company planted 7,410 seedlings each year.
   (a) Find the total number of seedlings planted in 12 years.
   (b) If 10% of the seedlings planted each year do not germinate, find the number of seedlings that germinated after 12 years.

B8 1 1 1 7 Determine perfect squares and square roots

Let us use multiples of numbers to determine how to identify perfect square numbers.

The first 4 multiples of 2 are: 2, 4, 6, 8.

2	4	6	8
↓	↓	↓	↓
2×1	2×2	2×3	2×4
	↓		
	2^2		

Therefore, 4 is a perfect square number.

The first 4 multiples of 3 are: 3, 6, 9, 12, ...

3	6	9	12
↓	↓	↓	↓
3×1	3×2	3×3	3×4
		↓	
		3^2	

Therefore, 9 is a perfect square number.

The first 4 multiples of 4 are: 4, 8, 12, 16, ...

4	8	12	16
↓	↓	↓	↓
4×1	4×2	4×3	4×4
			↓
			4^2

Therefore, 16 is a perfect square number.

From the explanation above, the first ten perfect square numbers are

1×1	2×2	3×3	4×4	5×5
↓	↓	↓	↓	↓
1	4	9	16	25

6×6	7×7	8×8	9×9	10×10
↓	↓	↓	↓	↓
36	49	64	81	100

Let us also use the idea of addition of odd numbers to determine perfect square numbers.

$1 + 3 = 4$

$1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 = 16$

$1 + 3 + 5 + 7 + 9 = 25$

$1 + 3 + 5 + 7 + 9 + 11 = 36$

$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$ etc.

We can use the illustration on addition of odd number for perfect square numbers to also determine the square root of perfect square numbers.

We know that $1 + 3 = 4$, we added two numbers to get 4, it means that 2 is the square root of 4.

We know that $1 + 3 + 5 = 9$, we added three numbers to get 9, it means that 3 is the square root of 9.

We know that $1 + 3 + 5 + 7 = 16$, we added four numbers to get 16, it means that 4 is the square root of 16.

EXERCISE 139 Date

1. Write the first 15 perfect square numbers.
2. Determine the square root of the following numbers.
   (a) 64 (b) 81
   (c) 100 (d) 196
   (e) 225

BEST BRAIN

B8.1.1.1.8 Use the knowledge on sets to solve problems

Look at the numbers: 3,6,9,12,15 and 18

We can see that the numbers represent the first 6 multiples of 3. We can write these numbers as a set of first 6 multiples of 3 as {3,6,9,12,15,18}.

Consider the names: Toyota, Benz, Peugeot, Nissan

We can see that the names represent brands of vehicles. We can write these names as a set as {Toyota, Benz, Peugeot, Nissan}.

A set is a well-defined collection of objects or items or numbers. The objects or items in a set are called members or elements. Sets are always written in curly brackets { }.

Example 466

Describe the set in words {Sunyani, Cape Coast, Kumasi, ...}

Solution

Set of capital towns of Ghana

EXERCISE 140 Date

Describe the following sets in words

a) {1,3,5,7,9}

b) {2,3,5,7,11,13,17}

c) {right angle triangle, isosceles triangle, scalene triangle}

d) {Hilla Lyman, J.J Rawlings, J.A Kuffour, J.A Mills,...}

e) {5,10,15,...}

Describing and writing sets

We often use capital letters to represent a set.

Look at the set P = {Rawlings, Kufuor, Mills, ...}

We can see that the set P is describing formal presidents of Ghana.

We can also use symbol $\in$ to describe whether a member belongs to a set or not. From set P, we can say that Rawlings $\in P$

This is read as Rawlings is a member of set P

We can also write Obasanjo $\notin P$, this means that Obasanjo is not a member of a set P.

Example 467

Write 7 $\notin$ {factors of 6} in full

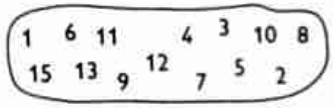
Solution

7 is not a member of the set of factors of 6

EXERCISE 141 Date

1. Write the following statements in full
   (a) $5 \in$ {multiples of 5}
   (b) $2 \in$ {Prime numbers}
   (c) $21 \in$ {multiples of 8}
   (d) $9 \in$ {positive whole numbers less than 11}

2. Use the elements in the space to write following sets



   a) M = {odd numbers}
   b) N = {perfect square numbers}
   c) R = {multiples of 4}

BEST BRAIN

Types of set

A set is described depending on the number of elements it has.

Consider the set P = {odd numbers less than 10}.

The elements of set P = {1,3,5,7,9}

P is an example of a finite set since its first and last elements are known.

Consider another set Q = {odd numbers}

The elements of set Q = {1,3,5,7,...}

Q is an example of an infinite set since odd numbers continues indefinitely.

Consider the set R = {1,3,4,9,10}

We can write the elements of R which is a multiple of 5 as Y = {10}

This set has only one element. A set that has only one element is called a unit set.

Consider the set M = {3,4,5,6}

The elements of M which is/are divisible by 7 is N = { }

The set N has no element because none of the elements in M is divisible by 7. A set that has no element is called an empty set.

An empty set is written as { } or ∅.

EXERCISE 142 Date

Describe each of the following sets as finite, infinite, unit or empty set.

(a) A = {2, 4, 6, 8, 10}

(b) B = {Joseph}

(c) C = {1, 3, 5, 7, ...}

(d) D = {5, 10, 15, ..., 500}

(e) E = {Perfect square in 2, 3, 5, 7}

(f) F = {7, 8, 9, 10, ...}

Equal and equivalent set

Look at the two sets

X = {2, 7, 8, 13}

Y = {8, 7, 13, 2}

We can observe that the two sets have the same elements and the same number of elements.

Two sets are equal if they have the same elements and the same number of elements.

Consider another two sets

M = {a, b, c, d, e}

N = {2, 3, 4, 5, 6}

We can observe that the two sets have the same number of elements but not the same elements.

Two or more sets are called equivalent set if they have the same number of elements.

EXERCISE 143 Date

1. Which of the following sets are equal?
   A = {x, y, z}, B = {3, 5, 7}, C = {y, z, x}, D = {5, 7, 9}, E = {7, 5, 9}

2. Which of the following sets are equivalent?
   A = {x, y, z}, B = {3, 5, 7, 9}, C = {y, z, x}, D = {1,5,7,9}, E = {7,5,9}

3. The set A = {3,5,7,11} and B = {3,11,a,7} are equal, find a.

Subset

Consider the set P = {a,b,c} and the set Q = {b,c}.

We can observe that all the elements of set Q are also in set P. We say that Q is a subset of P.

This is written using mathematical symbol as $Q \subset P$.

We can write all the sets that are subset of P as {},{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}.

Set P has 3 elements and 8 subsets.

The set A = {x,y} has subsets { }, {x}, {y}, {x,y}.

The set has 2 elements and 4 subsets. We use the formula 2^n to determine the number of subsets of a set.

EXERCISE 144 Date

1. Write down all the subsets of each of the following sets.
   (a) P = {e, f}
   (b) Q = {1, 2, 3}

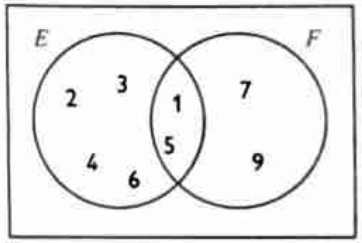
2. State the number of subsets of the following sets.
   (a) X = {d}
   (b) Y = {8, 9, 10}
   (c) Z = {e, f, g, h}

Intersection of sets

Consider the sets E = {1,2,3,4,5,6} and F = {1, 5, 7, 9}. We can observe some elements that are in E are also in F. Elements that can be found in both sets are called intersection elements. We use a mathematical symbol (∩) to represent intersection of sets.

From set E and F, we can write their intersection as: $E \cap F$ = {1,5}

We can also describe the intersection of two or more sets using a Venn diagram. A Venn diagram is an illustration that uses circles to show the common or difference between groups of things as shown below.



Example 468

Two sets A and B are defined as
A = {multiples of 3 less than 18} and
B = {prime numbers less than 15}.
(a) List all the elements of set A and B;
(b) Find $A \cap B$.

Solution

(a) A = {3,6,9,12,15} and B = {2,3,5,7,11,13}
(b) $A \cap B$ = {3}

Example 469

Given that M = {factors of 12} and N = {multiples of 2 less than 14}

(a) List all the elements of set M and N;

(b) Find $M \cap N$;

(c) Find the number of elements in $M \cap N$;

(d) Represent M and N on a Venn diagram.

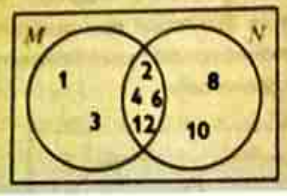
Solution

(a) $M = \{1,2,3,4,6,12\}$ and $N = \{2,4,6,8,10,12\}$

(b) $M \cap N = \{2,4,6,12\}$

(c) $n(M \cap N) = 4$

(d)



EXERCISE 145 Date

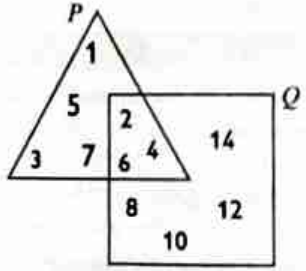
1. $M = \{5,6,7,8,9\}$ and $N = \{1,3,5,7,9,11\}$ are two sets. Find $M \cap N$.

2. $G = \{e,f,a,b,c\}$ and $H = \{d,e,f,g,h\}$ are two sets.
   (i) Find $G \cap H$;
   (ii) Represent G and H on a Venn diagram.

3. Two sets C and D are defined as
   C = {factors of 18} and
   D = {multiples of 6 less than 20}.
   (i) List of the elements of set C and D;
   (ii) Find $C \cap D$.

4. Given that S = {prime numbers less than 20} and $T = \{1,2,3,5,7,9,11\}$
   (a) List of the elements of set S;
   (b) Find $S \cap T$;
   (c) Find the number of elements in $S \cap T$;
   (d) Represent S and T on a Venn diagram.

5. Consider the diagram



   (a) List the elements of set P and Q;
   (b) Write $P \cap Q$;
   (c) Write the number of elements in Q.

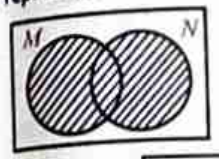
BEST BRAIN

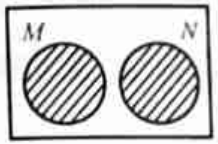
Union of sets

Consider the sets C = {5,6,7,8,9} and D = {7, 9, 11} The union of the two sets represented mathematical by the symbol ($\cup$) is the elements in all the two sets without repeating elements that are in both sets. From set C and D, the union is written as $C \cup D = \{5,6,7,8,9,11\}$.

We can also describe the union of two or more sets using a Venn diagram.

The shaded region in the figures below represent $M \cup N$.





Example 470

Two sets A and B are defined as
A = {factors of 8} and
B = {prime numbers less than 10}

(a) List the elements of A and B.

(b) Find
   (i) $A \cap B$
   (ii) $A \cup B$

Solution

(a) $A = \{1,2,4,8\}$ and $B = \{2,3,5,7\}$

(b) (i) $A \cap B = \{2\}$
   (ii) $A \cup B = \{1,2,3,4,5,7,8\}$

Example 471

Given that F = {whole numbers from 10 to 15} and G = {factors of 20}

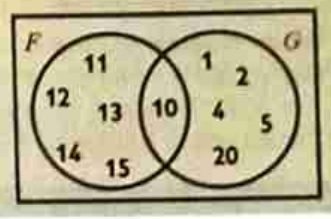
(a) List the elements of $F \cup G$

(b) Represent F and G on a Venn diagram

Solution

(a) $F = \{10,11,12,13,14,15\}$ and
$G = \{1,2,4,5,10,20\}$
$F \cup G = \{1,2,4,5,10,11,12,13,14,15,20\}$

(b)



EXERCISE 146 Date

1. Two sets X and Y are defined as
   X = {factors of 6} and Y = {factors of 8}
   (a) List the elements of X and Y
   (b) Find (i) $X \cap Y$ (ii) $X \cup Y$

2. Two sets M and N are defined as
   M = {multiples of 2 less than 10} and
   N = {multiples of 3 less than 12}
   (a) List the elements of M and N
   (b) Find (i) $M \cap N$ (ii) $M \cup N$

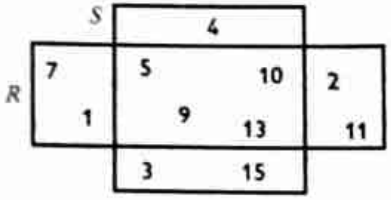
3. Given that $C = \{1,3,4,5\}$ and $D = \{1,5,9,11\}$
   (a) Find $C \cup D$.
   (b) Represent C and D on a Venn diagram.

4. Given that $P = \{7,9,11,13,15\}$ and Q = {multiples of 3 less than 18}

BEST BRAIN

(a) List the elements of Q.
(b) Find $P \cup Q$.
(c) Find $P \cap Q$.

5. S and R are two intersecting sets as shown.



(a) List the elements of S and R.
(b) Find (i) $S \cap R$ (ii) $S \cup R$

Universal set

A universal set denoted by the symbol (U) is a set that has elements in all the related sets in a given problem without repeating any of the elements. If P and Q are two sets, such that $P = \{1, 2, 3\}$ and $Q = \{2, 3, 4\}$ then the universal set will have its least elements as $U = \{1, 2, 3, 4\}$.

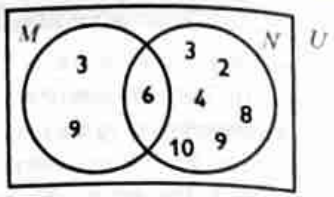
Example 472

M and N are subsets of the universal set U.
If $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$M = \{\text{multiples of 3 less than 12}\}$ and
$N = \{\text{multiples of 2 less than 12}\}$
(i) List the elements of M and N.
(ii) Find the number of elements in $M \cup N$.
(iii) Illustrate U, M and N on a Venn diagram.

Solution

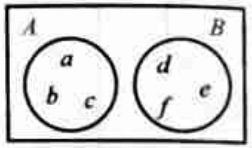
(i) $M = \{3, 6, 9\}$ and $N = \{2, 4, 6, 8, 10\}$
(ii) $M \cup N = \{2, 3, 4, 6, 8, 9, 10\}$
Therefore the number of elements in $M \cup N$ is 7*a*.

(iii)



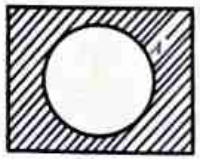
Disjoint set

Consider the set $A = \{a, b, c\}$ and $B = \{d, e, f\}$ We observe that $A \cap B = \{\ \}$ or $\varnothing$. We say that the set A and B are disjoint. This can be represented on a Venn diagram as shown.



Complement of a set

Consider the universal set $U = \{1, 2, 3, 4, 5\}$ and a subset $A = \{2, 4\}$. The complement of the set A denoted by $A' = \{1, 3, 5\}$. Complement of a set say A is/are the elements which belong to the universal set but do not belong to A.
The shaded region in the diagram illustrates the complement of A.



Example 473

P is the subset of the universal set U, where $U = \{3, 4, 5, 6, 7, 8, 9\}$ and $P = \{\text{multiples of 3 less than 10}\}$. Find the set P', the complement of P

BEST BRAIN

Solution

$U = \{3, 4, 5, 6, 7, 8, 9\}$
$P = \{3, 6, 9\}$
$P' = \{4, 5, 7, 8\}$

Example 474

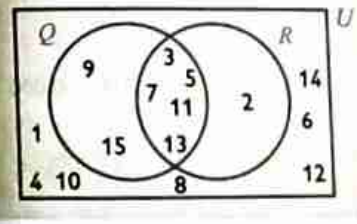
Q and R are subset of the universal set U. Given that
$U = \{\text{numbers from 1 to 15}\}$,
$Q = \{\text{odd numbers}\}$ and $R = \{\text{prime numbers}\}$
(a) List the elements in Q and R.
(b) Find (i) $Q \cup R$ (ii) $(Q \cup R)'$
(c) Illustrate Q, R and U on a Venn diagram.

Solution

(a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
Note that the elements in Q and R cannot go beyond 15 because they are subset of U.
$Q = \{3, 5, 7, 9, 11, 13, 15\}$
$R = \{2, 3, 5, 7, 11, 13\}$

(b) (i) $Q \cup R = \{2, 3, 5, 7, 9, 11, 13, 15\}$
(ii) $(Q \cup R)' = \{1, 4, 6, 8, 10, 12, 14\}$

(c)



EXERCISE 147 Date

1. C is the subset of the universal set U, where
$U = \{11, 12, 13, 14, 15, 16\}$ and
$C = \{12, 13, 14, 15\}$
Find the set C', the complement of C

2. F is the subset of the universal set U, where
$F = \{12, 15, 18\}$ and
$U = \{\text{multiples of 3 less than 20}\}$
Find F', the complement of F

3. X and Y are subset of the universal set U, where $U = \{\text{factors of 36}\}$, $X = \{1, 2, 4, 6\}$ and $Y = \{\text{multiples of 3 less than 18}\}$
(a) List the elements in U and Y
(b) Find
(i) $X \cup Y$
(ii) $X \cap Y$
(iii) $(X \cap Y)'$
(c) Illustrate X, Y and U on a Venn diagram.

4. G and H are subset of the universal set U Given that
$U = \{1, 2, 3, \ldots, 17\}$, $G = \{\text{even numbers}\}$ and
$H = \{\text{prime numbers}\}$
(a) List the elements in G and H
(b) Find
(i) $G \cup H$;
(ii) $(G \cup H)'$.
(c) Illustrate G, H and U on a Venn diagram.

BEST BRAIN

STRAND 1.2

NUMBER OPERATIONS

B8.1.2.1.1A Multiply given numbers by multiples of ten

In this section, we shall use our knowledge from Basic 7 on how to multiply whole numbers and decimal numbers to multiply numbers that are multiples of 10 by another number or decimal fractions.

Example 475

Find 721 × 1,000

Solution

721 × 1000 = 721,000

Example 476

Find the product of 516 and 700

Solution

$516 \times 700 = (516 \times 7) \times 100$

$= 3612 \times 100$

$= 361,200$

$$\begin{array}{r} 516 \\ \times \quad 7 \\ \hline 3612 \end{array}$$

Example 477

Find 68.082 × 10,000

Solution

Recall that $68.082 = \frac{68082}{1000}$

$68.082 \times 10,000 = \frac{68082}{1000} \times 10,000$

$= 68082 \times 10$

$= 680,820$

Example 478

Find $373.078 \times \frac{1}{1000}$

Solution

Recall that $373.078 = \frac{373.078}{1000}$

$373.078 \times \frac{1}{1000} = \frac{373.078}{1000} \times \frac{1}{1000}$

$= \frac{373078 \times 1}{1000 \times 1000}$

$= \frac{373078}{1000000}$

$= 0.373078$

EXERCISE 148 Date

Simplify the following

(a) 458 × 1,000 (b) 318 × 20,000

(c) 39.515 × 100 (d) $7.6 \times \frac{1}{100}$

(e) 82.0702 × 100,000

(f) $1217.368 \times \frac{1}{1000}$

B8.1.2.1.1B Divide given whole number by a multiple of ten

In this section, we shall use our knowledge from Basic 7 on how to divide whole numbers and decimal numbers to divide whole number, decimal or fractions by a multiply of 10.

Example 479

Find 7,421 ÷ 1,000

Solution

$7,421 \div 1,000 = 7421 \div \frac{1000}{1}$

$= 7421 \times \frac{1}{1000}$

$= \frac{7421}{1000}$

$= 7.421$

Example 480

Find $124.57 \div \frac{1}{100}$

Solution

$124.57 \div \frac{1}{100} = \frac{12457}{100} \div \frac{1}{100}$

$= \frac{12457}{100} \times \frac{100}{1}$

$= 12,457$

Example 481

Find 15.036 ÷ 1,000

Solution

$15.036 \div 1,000 = \frac{15036}{1000} \div \frac{1000}{1}$

$= \frac{15036}{1000} \times \frac{1}{1000}$

$= \frac{15036}{1000000}$

$= 0.015036$

Example 482

Find $93.15 \div \frac{1}{1000}$

Solution

$93.15 \div \frac{1}{1000} = \frac{9315}{100} \div \frac{1}{1000}$

$= \frac{9315}{100} \times \frac{1000}{1}$

$= 9315 \times 10$

$= 93,150$

EXERCISE 149 Date

Find the following.

(a) 23,000 ÷ 100 (b) 216 ÷ 10,000

(c) 153.17 ÷ 100 (d) $802 \div \frac{1}{100}$

(c) $715.024 \div \frac{1}{10}$ (d) $25.0927 \div \frac{1}{10,000}$

B8.1.2.1.2 Apply mental strategies and number properties to solve problems

In this section, we shall solve problems mentally using our knowledge on the four basic operations and number properties.

Idea of halving and doubling

In Basic 7, we solved problems mentally using number properties. In this section, we shall use the same concept to solve problems mentally.

Example 483

Find 36 × 15 mentally using the idea of halving and doubling.

Solution

Double of 15 is 30 and half of 36 is 18
Therefore, $36 \times 15 = 18 \times 30$
$= 18 \times 3 \times 10$
$= 54 \times 10$
$= 540$

Example 484

Find 75 × 52 mentally using the idea of halving and doubling.

Solution

Double of 75 is 150 and half of 52 is 28
Therefore, $75 \times 52 = 150 \times 28$
$= 150 \times (30 - 2)$
$= 150 \times 30 - 150 \times 2$
$= 4500 - 300$
$= 4,200$

Example 485

Workout 705 + 825

Solution

$705 = 700 + 5$
$825 = 700 + 125$
We have $700 + 700 + 125 + 5$
(This is friendlier to add)
$700 + 700 + 125 + 5$
$= 1400 + 130$
$= 1,530$

Example 486

Workout 1350 + 2050

Solution

$1350 = 1000 + 350$
$2050 = 1000 + 1000 + 50$
We have $1000+1000+1000+350 + 50$
(This is friendlier to add)
$1000+1000+1000+350 + 50$
$= 3000 + 400$
$= 3,400$

EXERCISE 150 Date.....

Find the following mentally using the idea of halving and doubling.

a) 5×324 b) 64×15
c) 50×415 d) 312×150
e) 80×125 f) 702×450

Adding by starting at the larger number and counting up by friendly jumps

Example 487

Workout 300 + 1230

Solution

For 300 + 1230, start from 1230 and count up three hundreds 1330, 1430, 1530.
So, 1230 + 300 = 1530.

Example 488

Workout 5449 + 401

Solution

For 5449 + 401, start from 5449 and count up four hundreds 5549, 5649, 5749, 5849. Then, add the 1 to get 5850.
So, 5449 + 401 = 5850.

Adding by decomposing the number(s) and then doing 'friendly jumps'.

Example 489

Workout 163 + 220

Solution

Instead of 163 + 220, think; 100 + 63 + 200 + 20
Now, 100 + 200 + 63 + 20,
which 300 + 83 = 383
So, 163 + 220 = 383

Example 490

Workout 2510 + 1405

Solution

2000 + 500 + 10 + 1000 + 400 +5
2000+1000+500+400+10+5
Now we have, 3000 + 900 + 15 = 3,915
So, 2510 + 1405 = 3,915

Adding by compensation

Example 491

Workout 780 +155

Solution

Rather than adding 780 +155, add 20 to 780 to create the addition 800 + 155, which is easy to add
Now, 800 + 155 = 955.
Then subtract 20 from 955 and get 935, because you added 20 more than you should have.
So 780 +155 = 935.

Example 492

Workout 876 + 130

Solution

Rather than adding 876 + 130, subtract 6 from 876 to create the addition 870 + 130, which is 1,000. Then add 6 to this answer, because you added 6 less than you should have.
So, 876 + 130 = 1006.

Subtracting Strategies
Subtracting by counting down (when subtracting a small number)

Example 493

Workout 1555 – 405

Solution

For 1555 – 405, start at 1555 and count back 405 in hundreds ...1455, 1355, 1255, 1155, then in ones 1155 – 5 = 1150.
So, 1555 – 405 = 1150.

Example 494

Workout 780 – 155

Solution

For 780 – 155, start at 780 and count back 155 in hundred ... 680, then in tens 680 – 50, then in ones 630 – 5 = 625.
So, 780 – 155 = 625.

Example 495

Workout 876 – 130

Solution

For 876 – 130,
start at 876 and count back 130 in hundred ... 776, then in tens 776 – 30 = 746.
So, 876 – 130 = 746.

Example 496

Workout 3400 – 2150

Solution

For 3400 – 2150
Start at 3400 and count back 2150 in thousands ... 2400, 1400, then in hundreds 1300, then finally subtract the 50.
Thus, 1300 – 50 = 1250.
So, 3400 – 2150 = 1250.

Subtracting by decomposing the second number into friendlier numbers

Example 497

Workout 163 – 121

Solution

Rather than subtracting 163 – 121, decompose the second number, into either 120 + 1 or 100 + 20 +1 and write (or think mentally) 163 – 120 – 1 Or 163 – 100 – 20 – 1
So 163 – 121 = 42 by subtracting one number at a time.

Example 498

Workout 700 – 345

Solution

Rather than subtracting 700 – 345, decompose the second number, into either 300 + 45 or 100 + 100 + 100 + 45 and write (or think mentally) 700 – 300 – 45.
So 700 – 345 = 400 – 45 = 355
i.e. by subtracting one number at a time.

Subtracting by "constant difference", i.e. by adding or subtracting the same amount from each number to create friendlier combinations

Example 499

Workout 163 – 129

Solution

Rather than subtracting 163 – 129, add 1 to 129 to create the subtraction 163 – 130, which is an easy combination to subtract and gives 33
Then adjust the answer (because you subtracted 1 more than you should have) by adding 1 to the answer. 33 + 1 = 34, So 163 – 129 = 34.

Example 500

Workout 550 – 531

Solution

Rather than 550 – 531, subtract 1 to create the subtraction 550 – 530, which is an easy combination to subtract and gives 20.
Then subtract 1 from this answer to get 19, because you subtracted 1 less than you should have.
So 550 – 531 = 19.

Subtracting by turning a subtraction into an addition

Example 501

Workout 185 – 155

Solution

Instead of 185 – 155, think! 155 + what = 185?
That will be 155 + 30 = 185
Therefore, 185 – 155 = 30.

Example 502

Workout 340 – 290

Solution

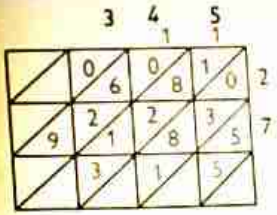
Instead of 340 – 290, think! 290 + what = 340?
That will be 290 + 50 = 340
Therefore, 340 – 290 = 50.

Multiplication Strategies
Lattice Method

Example 503

Workout 344 × 27

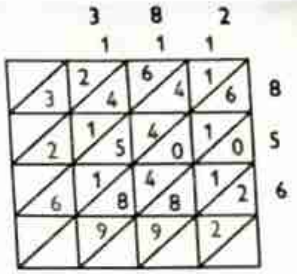
Solution



So, 345 × 27 = 9315

Example 504

Workout 382 × 856



So, 382 × 856 = 326,992

Expand and Box" method

Example 505

Workout 448 × 2

Solution

×	400	40	8
2	800	80	16

Now, 800 + 80 + 16 = 896
So, 448 × 2 = 896

Example 506

Workout 2,431 × 106

Solution

	2,000	400	30	1	
6	12,000	2,400	180	6	14,586
100	200,000	40,000	3,000	100	243,100

14,586 + 243,100 = 257,687
So, 2,431 × 106 = 257,686

Multiplying by 10, 100, and 1000, as well as multiples of 10, 100, and 1000

Example 507

Workout 45 × 100

Solution

When you are faced with the problem 45×100, you need to understand that the place values increase by 2 places, to make the product 4500. That is, just add the two zeros to the 45 to get 4500.

Example 508

Workout 425 × 1000

Solution

Similarly, when multiplying a problem like 425 ×1000, the place values increase by 3 places to make 425000.

Partial Products Strategy

In partial products, the equation is set up like in traditional long multiplication, but the way we multiply is different.

Example 509

Workout 125 × 3

Solution

For the problem 125 × 3, we first multiply 3 × 100 to make 300.
Then we multiply 3×25 to make 75.
Now we add the 300 and 75 together to make 375.

Example 510

Workout 2500 × 4

Solution

For the problem 2500 × 4, we first multiply 4 × 25 to make 100.
Now we add the two zeros to make 10,000.

Division Strategies

Distributive property

Example 511

Workout 1554 ÷ 6

Solution

(1200 ÷ 6) + (300 ÷ 6) + (54 ÷ 6) =
200 + 50 + 9 = 259
So, 1554 ÷ 6 = 259

Example 512

Workout 184,554 ÷ 9

Solution

(180,000 ÷ 9) + (4,500 ÷ 9) + (54 ÷ 9) =
= 20,000 + 500 + 6 = 20,506
So, 184,554 ÷ 9 = 20,506

Area Model Division

Example 513

Workout 825 ÷ 5

Solution

So, 825 ÷ 5 = 165

Example 514

Workout 728 ÷ 14

Solution

Let's start by breaking down 728 into 700 and 28.
Dividing 700 by 14 gives us 50, and 28 divided by 14 is 2.

728 ÷ 14 = ?

50 + 2

	728	28
14	- 700	- 28
	28	0

728 ÷ 14 = 50 + 2
= 52

Now, let's add these two to get the quotient 50 + 2, which gives us 52.

Big 7 Division Strategy

Example 515

Workout 875 ÷ 6

Solution

6	875	
	-600	100 × 6
	275	
	- 60	10 × 6
	215	
	- 120	20 × 6
	95	
	- 60	10 × 6
	35	
	- 30	+ 5 × 6
	5	145 R 5

So, 875 ÷ 6 = 145 Remainder 5

Example 516

Workout 864 ÷ 3

Solution

3	864	
	- 600	3 × (200)
	264	
	- 180	3 × (60)
	84	
	- 60	3 × (20)
	24	
	- 24	3 × (8)
	0	

Now, 200 + 60 + 20 + 8 = 288
So, 864 ÷ 3 = 288

B8.1.2.1.3 Apply mental strategies and number properties to solve word problems

In this section, we shall solve some real life problems mentally using the four basic operations and number properties.

Example 517

A farmer harvested 7,459 pineapples. She sold some of the pineapples to a trader and was left with 5,952. Find the number of pineapples sold to the trader.

Solution

Number sentence: $7459 - \Delta = 5952$

$\Delta = 7459 - 5952$

$\Delta = 1507$

Therefore, the number of pineapples sold is 1,507

Example 518

Mr. Appiadu a Basic 7 teacher in Ekumfi M/A Basic School gave some exercise books to his two best students, Ayeki and Baba. Baba received twice as many books as received by Ayeki. If Baba had 22 books, find the total number of books received by the learners.

Solution

Number sentence: $\Delta = \frac{1}{2} \times 22 + 22$

$\Delta = \frac{1}{2} \times 22 + 22$

$= 11 + 22$

$= 33$

Therefore, total number of books received by the learners is 33

Example 519

A man had 42 fowls, he sold them to some buyers equally. If each of the buyers received 14 fowls, how many buyers bought the fowls?

Solution

Number sentence: $42 \div \Delta = 14$

$\Delta = 42 \div 14$

$\Delta = 3$

Therefore, the man sold the fowls to 3 buyers.

EXERCISE 151 Date

1. A man gave some money to his 3 children Kofi, Abena and Ama. Kofi received twice as Ama and Abena received twice as Kofi. If Ama had GH¢10.00. Find the money the man gave to his children.

2. A trader bought some number of bags of millet, she sold 52 bags of the millet to a friend and was left with 128 bags. How many bags of millet did the trader buy?

3. A shop owner had 20 radio sets, he sold each radio set for GH¢46.00. Find the total amount received by the shop owner after selling all the radio sets.

4. A woman had 52 tubers of yam, she sold them equally to some customers. If each customer had 13 tubers of yam, how many customers bought the yam?

B8.1.2.1.4 Perform addition and subtraction of integers and decimal numbers

In this section, we shall use our knowledge on addition and subtraction of integers from Basic 7 to add and subtract integers up to 9-digit and decimal numbers using expanded form and place value.

Example 520

Find the sum of 3,152,301 and 813,413

Solution

Using expanded form

$$\begin{array}{rl} 3\,152\,301 = & 3{,}000{,}000 + 100{,}000 + 50{,}000 + 2{,}000 + 300 + 00 + 1 \\ +\ 813\,413 = & \quad 800{,}000 + 10{,}000 + 3{,}000 + 400 + 10 + 3 \\ \hline 3\,965\,714 = & 3{,}000{,}000 + 900{,}000 + 60{,}000 + 5{,}000 + 700 + 10 + 4 \end{array}$$

Using place value method

M	H-th	T-th	Th	H	T	O
3	1	5	2	3	0	1
+	8	1	3	4	1	3
3	9	6	5	7	1	4

Example 521

Subtract 7,777 from 18,834

Solution

Using expanded form

$$\begin{array}{rl} 18834 = & 10{,}000 + 8{,}000 + 800 + 30 + 4 \\ -\ 7777 = & \quad -7{,}000 - 700 - 70 - 7 \\ \hline 11057 = & 10{,}000 + 1{,}000 + 000 + 50 + 7 \end{array}$$

Using place value method

T-th	Th	H	T	O
1	8	8	3	4
−	7	7	7	7
1	1	0	5	7

Example 522

Find the sum of 916.521 and 82.615

Solution

Expanded form

$$\begin{array}{rl} 916.521 = & 900 + 10 + 6 + \frac{5}{10} + \frac{2}{100} + \frac{1}{1000} \\ +\ 82.615 = & \quad 80 + 2 + \frac{6}{10} + \frac{1}{100} + \frac{5}{1000} \\ \hline & = 900 + 90 + 8 + \frac{11}{10} + \frac{3}{100} + \frac{6}{1000} \\ 999.136 = & 900 + 90 + 9 + \frac{1}{10} + \frac{3}{100} + \frac{6}{1000} \end{array}$$

Place value chart

H	T	O	t	h	th
9	1	6	5	2	1
+	8	2	6	1	5
9	9	9	1	3	6

Example 523

Find the difference between 451 928 and 780.514

Solution

Expanded form

$780.514 = 700 + 80 + 0 + \frac{5}{10} + \frac{1}{100} + \frac{4}{1000}$

$-451.928 = 400 - 50 - 1 - \frac{9}{10} - \frac{2}{100} - \frac{8}{1000}$

$328.136 = 300 + 20 + 8 + \frac{5}{10} + \frac{8}{100} + \frac{6}{1000}$

Place value chart

H	T	O	t	h	th
7	8	0	5	1	4
−4	5	1	9	2	8
3	2	8	5	8	6

EXERCISE 152 Date

1. Add 51,928 and 8,271 using expanded form
2. Find the sum of 741,029 and 61,829 using place value chart.
3. Subtract 8,271 from 61,271 using expanded form.
4. Find the difference between 821.921 and 71,819 using place value chart.
5. By how much is 519.809 bigger than 45.918
6. Find the sum of 71.917 and 827.927

B8.1.2.1.5A Multiply multi-digit numbers by 1, 2 and 3-digit numbers

In this section, we shall perform multiplication of multi-digit numbers by 1, 2 or 3-digit numbers using expand and box method, vertical place value method, lattice method or distributive method.

Example 524

Find 284 × 73

Solution

Using expand and box method

200	80	4	
200×70=14,000	80×70=5,600	4×70=280	70
200 × 3 = 600	80 × 3 = 240	4 × 3 = 12	3

284×731

= 14,000 + 5,600 + 280 + 600 + 240 + 12

= 19,600 + 1,120 + 12

= 19,600 + 1,132

= 20,732

Using vertical place method

	H	T	O
	2	8	4
×		7	3
	8	5	2
+1	9	8	8
2	0	7	3	2

Example 525

Find 5,651 × 14

Solution

Using distributive method

5,651 × 14 = 5,651 × (10 + 4)

= 5651 × 10 + 5651 × 4

= 56510 + 22604

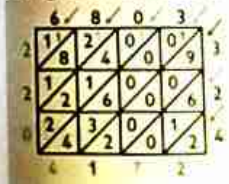
= 79,114

Example 526

Find 6,803 × 324 using lattice method and expand and box method.

Solution

Using Lattice method



Therefore 6,803 × 324

= 2,204,172

Using expand and box method

6,000	800	0	3	
6,000 × 300 = 1,800,000	800 × 300 = 240,600	0	3 × 300 = 900	300
6,000 × 20 = 120,000	800 × 20 = 16,000	0	3 × 20 = 60	20
6,000 × 4 = 24,000	800 × 4 = 32,000	0	3 × 4 = 12	4
1,944,000	259,200	0	972	

6,803 × 324 = 1,944,000 + 259,200 + 0 + 972

= 2,204,172

Date.....

EXERCISE 153

Find the following using any of the methods: expand and box method, vertical place value method, Lattice method or distributive method.

(a) 152 × 81 (b) 4,821 × 35 (c) 9,312 × 123

(d) 712 × 812 (e) 5,019 × 411 (f) 62 × 41,201

BEST BRAIN

B8 1.2.1.5B Determine basic division facts

In Basic 7, we solved some division facts using 2, 3, 4 and 7 based on our knowledge from Basic 6. In this section, we shall solve simple problems on division facts using 5, 6, 8, 9 and 11.

Example 527

Anthony is to pack 128 crates of eggs into boxes. Each box can contain 5 crates of eggs. How many crates of eggs will be left after the last box is filled.

Solution

The problem is a divisibility test of 5.
The last digit is 8
but 8 – 5 = 3
Therefore, it will be left with 3 crates of eggs after the last box is filled.

Example 528

Mrs. Afrim bought 6,176 oranges from a farm. If she decides to sell the oranges in groups of 6, how many oranges must be added to enable her sell the oranges in groups of 6?

Solution

The problem is a divisibility test of 6.
The last digit is 6 which is divisible by 2 and the sum of the digits 6 + 1 + 7 + 6 = 20 which is not divisible by 3
But when we add 4, the last digit will be divisible by 2 and the sum will also be divisible by 3.
Therefore, she must add 4 oranges.

Example 529

Kofi is to package 4,818 tubers of yam into boxes. If a box can contain 8 tubers of yam, how many tubers of yam will be left after the last package?

Solution

The problem is a divisibility test of 8
The last 3 digits is 818, let us use long division

$$\begin{array}{r} 8\overline{)818} \\ -816 \\ \hline 2 \end{array}$$

Therefore, it will be left with 2 tubers of yam after the last package.

Example 530

Mills shared 8,152 books among some learners such that each learner will have 9 books. How many books will be left?

Solution

The problem is a divisibility test of 9
The sum of the digits is 8 + 1 + 5 + 2 = 16
But 16 is not divisible by 9, but when we subtract 7 from 8,152 it will be left with 8145 where the sum of the digits is divisible by 9.
Therefore, there will be 7 books left

Example 531

Anita bought 6,148 litres of cooking oil from the market. She poured them into smaller bottles each of capacity 11 litres. Find the number of litres needed to fill the last bottle.

BEST BRAIN

Solution

The problem is a divisibility test of 11
6 − 1 + 4 − 8 = 1
But the result is not divisible by 11. When we add 1 to 6,148 the answer 6,149 which will be divisible by 11
Therefore, Anita needs 1 litre in order to fill the last bottle.

Using games and mental strategies are great ways to learn Math facts and develop mental calculation skills in a fun and easy way.
The following are some useful strategies

Dividing by 2
Whenever you are dividing by 2 you should break the number into friendly numbers.

Example 532
1726 ÷ 2

Solution

x	1	2	3	4	5	6	7	8	9
x × 2	2	4	6	8	10	12	14	16	18

1726 ÷ 2
1726 = 1600 + 120 + 6
1726 ÷ 2 = (1600/2) + (120/2) + (6/2)
800 + 60 + 3 = 863

Dividing by 4
For example 64 ÷ 4
Step 1 – Divide the number by 2:
64/2 = 32
Step 2 – Divide the number from step 1 by 2:
32/2 = 16

Dividing by 5
For example 1235 ÷ 5
Step 1 – Divide the number by 10:
1235/10 = 123.5
Step 2 – Multiply the number from step 1 by 2:
123.5 x 2 = 247

Dividing by 20
For example 1260 ÷ 20
Step 1 - Divide the number by 10:
1260/10 = 126
Step 2 – Divide the number from step 1 by 2:
126/2 = 63

Dividing by 25
For example 1250 ÷ 25
Step 1 – Divide the number by 100:
1250/100 = 12.50
Step 2 - Multiply the number from step 1 by 4:
12.5 x 4 = 50

Dividing by 40
Example 1280/40
Step 1 – Divide the number by 10:
1280/10 = 128
Step 2 – Divide the number from step 1 by 4:
128/4 = 32

Dividing by 50
Example 1250 ÷ 50
Step 1 – Divide the number by 100:
1250/100 = 12.5
Step 2 – Multiply the number from step 1 by 2:
12.5 x 2 = 25

EXERCISE 154 Date

1. Atinga is to pack 1,813 mangoes into containers. Each container can contain 5 mangoes. How many mangoes will be left after the last container is filled
2. A farmer harvested 5,266 watermelons. He wants to sell them among his customers such that each customer will have 6 watermelons. Find the number of watermelons he must add.
3. A mason bought 709 bags of cement to be used for making blocks. He uses 8 bags of cement each day. How many bags of cement will be left after the last day's work.
4. Aku wants to share GH¢4,142.00 among some friends such that each friend will get GH¢9.00 How much money must she add?
5. Kusi is to pack 9,445 books into boxes. Each box can contain 11 books. Find the number of books that will be left after the last package.

B8.1.2.1.6 Solve real life problems involving decimals

In this section, we shall use our knowledge on decimal numbers to solve real life problems using the four basic operations.

Example 533

A tank contains 516 litres. Quansah fetched 54.23 litres and Amina fetched 71.66 litres. How many litres of water is left in the tank?

Solution

Let the quantity of water left after = g

Number sentence: $g = 516 - 54.23 - 71.66$

$g = 516 - (54.23 + 71.66)$

$g = 516 - 125.89$

$g = 390.11$

54 23	516.00
+71.66	−125.89
125.89	390.11

Therefore 390.11 litres of water is left in the tank.

Example 534

A carpenter bought 90 pieces of wood with each piece having a length of 17.43 m. Find the total length of all the woods.

Solution

Let x = total length of wood bought

$x = 90 \times 17.43$

$= 90 \times \frac{1743}{100}$

$= 9 \times \frac{1743}{10}$

$= \frac{15687}{10}$

$= 1568.7$

Therefore the total length of wood is 1568.7 m

Example 535

A bag contains 7 boxes of apples. If the mass of the bag and the content is 15.62 kg and the mass of the bag only is 150 g. Find the mass of each box of apples.

Solution

Let the mass of a box of apples = n

Number sentence: $n = (15.62 \text{ kg} - 150 \text{ g}) \div 7$

But $150 \text{ g} = 150 \times \frac{1}{1000} \text{ kg} = \frac{150}{1000} \text{kg} = 0.15 \text{ kg}$

$n = (15.62 - 0.15) \div 7$

$= 15.47 \div 7$

$= \frac{1547}{100} \times \frac{1}{7}$

$= \frac{221}{100}$

$= 2.21 \text{ kg}$

Therefore a box of apples has a mass of 2.21 kg.

Example 536

Mr. Gyan budgeted to buy some items during the upcoming Christmas season. The table shows the budget he made for the season.

Activity	Cost (GH¢)
Food	712.33
Clothing	840.00
Utilities	283.75
Other expenditures	533.82

(a) Find the amount budgeted on food and utilities.

(b) If he spends three times the amount he budgeted for other expenditure, how much did he spend on other expenditure?

(c) How much was overspent on other expenses.

(d) How much more was budgeted on clothing than utilities?

Solution

(a) Amount budgeted on food and utilities
$= 712.33 + 283.75$
$= \text{GH¢}996.08$

(b) Amount spent on other expenditure
$= 3 \times 533.82$
$= 3 \times \frac{53382}{100}$
$= \frac{3 \times 53382}{100}$
$= \frac{160146}{100}$
$= 1601.46$

(c) Amount overspent on other expenses.
$= 1601.46 - 533.82 = \text{GH¢}1067.64$

(d) Amount budgeted on clothing than utilities
$= 840.00 - 283.75$
$= \text{GH¢}556.25$

EXERCISE 155 Date

1. By how much is 615.081 kg heavier than 302.965 kg?
2. A carpenter bought a wood of length 402.71 m. If he used 245.09 m of the wood, how many metres of the wood is left?
3. Mrs. Akam bought 201.634 litres of cooking oil. She decided to pour the cooking oil equally into 4 containers. How many litres of cooking oil will be in each container?
4. The mass of a man is 65.92 kg and the mass of his two children are 15.88 kg and 31.15 kg. What is the difference between the man's weight and the weight of the two children?
5. A farmers bought 11 bundles of rope. Each bundle of rope has a length of 344.67 m. Find the total length of the 11 bundles of ropes.
6. The table shows a budget made by the headmistress of Holy Spirit School for a given month.

Activity	Cost (GH¢)
White board markers	432.45
Teachers' notebooks	729.02
Utilities	355.64
Fuel	1,362.90

(a) How much more did she budget to spend on fuel than utilities?

(b) Find the total amount budgeted on white board marker and teachers' notebook.

(c) If she actually spent half of the amount budgeted on fuel, find the amount spent on fuel.

(d) How much did she save on fuel?

B8.1.2.1.7 Explain the laws of indices

In basic 7, we leant how to express a given number in index form. In this section, we shall use our knowledge from Basic 7 to solve problems in index form, using basic laws of indices.

First Law: Multiplication

To find the product $a^2 \times a^3$, we have

$a^2 \times a^3 = a \times a \times a \times a \times a$

$= a^5$

It can be seen that: $a^2 \times a^3 = a^{2+3}$

$= a^5$

If two integers x and y have the same base (say a) and are to be multiplied together, their indices are added.

$$a^x \times a^y = a^{x+y}$$

Second Law: Division

To find $a^5 \div a^2$, we have

$$a^5 \div a^2 = \frac{a^5}{a^2}$$

$$= \frac{a \times a \times a \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a}}$$

$$= a \times a \times a$$

$$= a^3$$

It can be seen that: $a^5 \div a^2 = a^{5-2}$

$= a^3$

If two integers x and y have the same base (say a) and are to be divided, their indices are subtracted.

$$a^x \div a^y = a^{x-y}$$

Third Law: Brackets

To find $(a^2)^3$, we have

$(a^2)^3 = a^2 \times a^2 \times a^2$

$= a^{2+2+2}$ (using law 1)

$= a^6$

It can be seen that: $(a^2)^3 = a^{2\times 3} = a^6$

If a term with an index is itself raised to an index, then the two indices are multiplied together.

$$(a^x)^y = a^{x \times y}$$

Fourth Law: Product or brackets

To find $(ab)^3$, we have

$(ab)^3 = ab \times ab \times ab$

$= a \times b \times a \times b \times a \times b$

$= a \times a \times a \times b \times b \times b$

$= a^3 \times b^3$

It can be seen that: $(ab)^3 = a^{1\times 3} \times b^{1\times 3}$

$= a^3 \times b^3$

If two positive integers multiplying have the same index then each integer is raised to the same index.

$$(ab)^y = a^y \times b^y$$

Property 1: Negative powers

To find $\frac{a^3}{a^5}$, we have

$$\frac{a^3}{a^5} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a}}{a \times a \times \cancel{a} \times \cancel{a} \times \cancel{a}}$$

$$= \frac{1}{a^2}$$

$\frac{a^3}{a^5} = a^{3-5}$ (using the second law)

$= a^{-2}$

It means that a negative power can be written as a fraction.

$$a^{-x} = \frac{1}{a^x}$$

BEST BRAIN

Property 2: Power of zero

To find $\frac{a^3}{a^3}$, we have

$$\frac{a^3}{a^3} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a}}$$

$$= \frac{1}{1}$$

$$= 1$$

$\frac{a^3}{a^3} = a^{3-3}$ (using the second law)

It means that any positive integer with an index of zero (0) is 1.

$$a^0 = 1$$

B8.1.2.1.8 Apply the laws of indices to solve problems

In the previous section, we used examples to explain and derived the laws of indices. In this section, we shall use the laws to solve problems. When solving problems involving indices we always use the order of operation: Parenthesis, Exponent, Division, Multiplication, Addition and Subtraction (PEDMAS).

Example 537

Simplify

(a) $y^5 \times y^3 \times y^2$ (b) $3^7 \times 3^3 \div 3^2$

(c) $a^7 \div a^4 \times a^{-3}$

Solution

(a) $y^5 \times y^3 \times y^2 = y^{5+3+2}$ (using the first law)

$= y^{10}$

(b) $3^7 \times 3^3 \div 3^2 = 3^7 \times 3^{3-2}$ (using PEDMAS)

$= 3^7 \times 3^1$

$= 3^{7+1}$ (using the first law)

$= 3^{10}$

(c) $a^7 \div a^4 \times a^{-3} = a^{7-4} \times a^{-3}$ (using PEDMAS)

$= a3 \times a^{-3}$

$= a3 + (-3)$ (using the first law)

$= a3^{-3}$

$= a^0$

$= 1$

Example 538

Simplify $(2^2 \times 3^3)^4 \div (2^3 \times 3^{10})$

BEST BRAIN

Solution

$(2^3 \times 3)^4 + (2^1 \times 3^{10}) = (2^3 \times^4 \times 3^1 \times^4) + (2^1 \times 3^{10})$
$= 2^{12} \times 3^{12} + (2^1 \times 3^{10})$
$= 2^{12} + 2^1 \times 3^{12} + 3^{10}$
$= 2^{12-1} \times 3^{12-10}$
$= 2^{11} \times 3^2$

Example 539

Evaluate (find the answer) the following.

(a) $(2^2 \times 7)^3 \div 7^3$ (b) $\frac{(2^3)^3 \times 5^8}{20^3}$

(c) $(24)^4 \div (2^{13} \times 3^3)$

Solution

(a) $(2^2 \times 7)^3 \div 7^3 = (2^2 \times^3 \times 7^1 \times^3) \div 7^3$
$= (2^6 \times 7^3) \div 7^3$
$= 2^6 \times 7^3 \div 7^3$
$= 2^6 \times 7^{3-3}$
$= 2^6 \times 7^0$
$= 16 \times 1$
$= 16$

(b) $\frac{(2^3)^3 \times 5^8}{20^3} = \frac{2^3 \times^3 \times 5^8}{(2 \times 2 \times 5)^3}$
$= \frac{2^9 \times 5^8}{2^3 \times 2^3 \times 5^3}$
$= \frac{2^9 \times 5^8}{2^6 \times 5^3}$
$= 2^{6-6} \times 5^{8-3}$
$= 1 \times 5^5$
$= 25$

(c) $(24)^4 + (2^{13} \times 3^3) = (2 \times 2 \times 2 \times 3)^4 + (2^{13} \times 3^3)$
$= (2^4 \times 2^4 \times 2^4 \times 3^4) + (2^{13} \times 3^3)$
$= 2^{4+4+4} \times 3^4 + (2^{13} \times 3^3)$
$= 2^{12} \times 3^4 \div (2^{13} \times 3^3)$
$(24)^4 \div (2^{13} \times 3^3) = 2^{12} \div 2^{13} \times 3^4 \times 3^{-3}$
$= 2^{12-13} \times 3^{4-3}$
$= 2^{-1} \times 3^1$
$= \frac{1}{2^1} \times 3$
$= \frac{3}{2}$

EXERCISE 156 Date.....

1. Simplify the following.
   (a) $x^4 \times x^3$
   (b) $y^{11} \div y^7 \times y^3$
   (c) $(3 \times 5)^4 \times 5^3 \times 3^4$
   (d) $(2 + 3)^4 \times 3^1 \times 2^1$
   (e) $b^8 \div b^3 \times b^{-3}$
   (f) $(5^3 \times 7^2)^4 \div 7^5 \times 5^{10}$
   (g) $2^1 \times 4^2 \times 8 \div 2^3$
   (h) $(36)^4 \times 2^5 \div 3^7$
   (i) $3^{-3} \times 5^{-7} \times (75)^3$

2. Evaluate the following.
   (a) $2^3 \div 2^7 \times 3$
   (b) $(3^2 \times 5)^4 \div 15^2$
   (c) $4^3 \times 7^5 \div (7^3 \times 4^2)$
   (d) $\frac{(2^2)^2 \times 4^3}{8^3}$
   (e) $\frac{7^5 \times 8^3}{28^3}$
   (f) $(2^{-4})^3 \times (2^3)^4$

BEST BRAIN

B8.1.2.1.9 Solve exponential equations

We know from powers of numbers that $4 = 2^2$. This means that when we write $2^2 = 2^x$ then $x = 2$ since both sides of the equation has a common base of 2.

Similarly, when we have $a^3 = 5^3$ then $a = 5$ since the index are the same.

These are examples of an exponential equation. In this section, we shall use this concept to solve problems involving an unknown bases or unknown exponents.

Example 540

Find the value of b if $m^4 \times m^3 = m^b$

Solution

$m^4 \times m^3 = m^b$
$m^{4+3} = m^b$
$m^7 = m^b$ (since the base are the same)
$7 = b$

Example 541

Find the value of x given that $2 \times 2^3 \div 2^2 = 2^{x-1}$

Solution

$2 \times 2^3 \div 2^2 = 2^{x-1}$
$2^1 \times 2^3 \div 2^2 = 2^{x-1}$
$2^4 \div 2^2 = 2^{x-1}$
$2^{4-2} = 2^{x-1}$
$2^2 = 2^{x-1}$ (equate exponents, since the bases are the same)
$2 = x - 1$
$2 + 1 = x - 1 + 1$
$3 = x$

Example 542

Find the value of a given that $5a^3 + 2a^3 - 3a^3 = 32$

Solution

$5a^3 + 2a^3 - 3a^3 = 32$
$7a^3 - 3a^3 = 32$
$4a^3 = 32$ (take the multiplicative inverse of 4 on both sides)
$\frac{1}{4} \times 4a^3 = \frac{1}{4} \times 32^8$
$a^3 = 8$
$a^3 = 2^3$
$a = 2$

Example 543

Find the value of m if $2^m + 50^0 = 17$

Solution

$2^m + 50^0 = 17$
$2^m + 1 = 17$
$2^m + 1 - 1 = 17 - 1$
$2^m = 16$
$2^m = 2^4$
$m = 4$

Example 544

$5 \times 25 \times 5^{3m} = 5^9$, find the value of m.

Solution

$5 \times 25 \times 5^{3m} = 5^9$
$5^1 \times 5^2 \times 5^{3m} = 5^9$
$5^{1+2+3m} = 5^9$
$5^{3+3m} = 5^9$
$5^{3+3m} = 5^9$ (equate exponents, since the bases are the same)
$3 + 3m = 9$

BEST BRAIN

$3 - 3 + 3m = 9 - 3$

$3m = 6$

$\frac{1}{3} \times 3m = \frac{1}{3} \times 6^{1}$

$m = 2$

Example 545

$3^{2x+1} = \frac{1}{27}$, find the value of x.

Solution

$3^{2x+1} = \frac{1}{27}$

$3^{2x+1} = \frac{1}{3^3}$

$3^{2x+1} = 3^{-3}$ (equate exponents, since the bases are the same)

$2x + 1 = -3$

$2x + 1 - 1 = -3 - 1$

$2x = -4$

$\frac{1}{2} \times 2x = \frac{1}{2} \times -4^{-2}$

$x = -2$

EXERCISE 157 Date

Find the value of the variables in each of the following equations.

(a) $2^{1} = 8$

(b) $3^{x+1} = 9$

(c) $a \times a^{2} \times a^{3} = 3^{4}$

(d) $x^{3} \times x^{4} + x^{3} = 2^{4}$

(e) $2^{4} + 8 \times 2^{x} = 32$

(f) $y \times y \times y^{2} = 16$

(g) $a^{2} + 2a^{2} + 3a^{2} = 24$

(h) $2b^{3} - 5b^{3} + 7b^{3} = 4$

(i) $3^{x} - 2 = 7$

(j) $3 \times 2^{m} + 2 = 14$

(k) $4 \times 8 \times 2^{2m} = 2^{7}$

(l) $5^{2x+1} = \frac{1}{25}$

B8.1.2.2.1 Solve real life problems involving powers of numbers

In this section, we shall use our knowledge on exponential equations to solve real life problems on powers of numbers.

Example 546

The population growth of fowls in Mr. Duah's farms is given by $(2^{x})^{3}$ where x is in months. Find the number of months needed to have a fowl population of 2^{12}.

Solution

$(2^{x})^{3} = 2^{12}$

$2^{3x} = 2^{12}$

$3x = 12$

$\frac{1}{3} \times 3x = \frac{1}{3} \times 12^{4}$

$x = 4$

Therefore it will take 4 months.

Example 547

The fuel consumption in litres by a country is given by 10^{3y+1} where y is in days. After how many days will the country consume a total fuel of 100^{14}.

Solution

$10^{3y+1} = 100^{14}$

$10^{3y+1} = (10^{2})^{14}$

$10^{3y+1} = 10^{2 \times 14}$

$10^{3y+1} = 10^{28}$

$3y + 1 = 28$

$3y + 1 - 1 = 28 - 1$

$3y = 27$

$\frac{1}{3} \times 3y = \frac{1}{3} \times 27^{9}$

$y = 9$

Therefore it will take 9 days to consume that litres of fuel.

EXERCISE 158 Date

1. The population growth of family in centuries is 9^{2a} where a is a century. After how many centuries will the total population of the family be 9^{4}?

2. The sale of apple phones in a country is given by 5^{x+3} where x is in months. After how many months will a sole dealer of apple phones in the country sell 5^{20}?

3. The rate at which a certain forest reserve is depleted in hectors is given by 3^{4x-3} where x is in years. Find the number of years after which the total hectors of forest depleted is 27^{3}?

4. Ants population is given by 10^{3x+1} where x is time in weeks. After how many weeks will the population of ants in a given colony be 1,000,000?

5. A substance decays by an expression 3^{y} where y is in years. Find the number of years needed for the substance to decay to a size of 9^{4}.

6. The number of cashew fruits obtained from a farm is given by 2×4^{3x}, where x is in years. Find the number of years, it will take to harvest 2^{11} cashew fruits.

STRAND 1.3 FRACTIONS, DECIMALS AND PERCENTAGES

B8.1.3.1.1 Review on equivalent, mixed and improper fractions.

In Basic 7, we reviewed the concept of equivalent, mixed and improper fractions. In this section, we shall use those ideas to reason through a few questions.

Example 548

Abena had a pizza that was divided into 8 equal slices. She ate 3 of them. Dauda has a pizza that is the same size, but he divided it into 4 equal slices. He ate 3 slices of his pizza. Who ate more pizza?

Solution

Let's identify the fraction to represent Abena and Dauda

Abena = $\frac{3}{8}$ Dauda = $\frac{3}{4}$

Now, using our knowledge of equivalent fractions, we know that since Abena's denominator is a double of Dauda's denominator, she also needs to double her numerator.

That is, $\frac{3}{4} = \frac{6}{8}$.

So, Abena should have eaten 6 slices to make it equal to the 3 slices that Dauda ate.

Therefore, we can say that Dauda ate more pizza.

Example 549

Akwesi made two pies that were exactly the same size. The first pie was a meat pie, which he cut into 6 equal slices. The second was a fish pie, which he cut into 12 equal pieces. Akwesi takes his pies to a party. People eat 3 slices of meat pie and 6 slices of fish pie. Did people eat more meat pie or fish pie?

Solution

Let's write the fractions for the two pies.

Meat pie = $\frac{3}{6}$ Fish pie = $\frac{6}{12}$

Again, let's use our knowledge of equivalent fractions to compare them.

Comparing the two fractions, we realise that they are equivalent. To be sure, we can reduce both fractions to the lowest form

$\frac{3}{6} = \frac{1}{2}$

$\frac{6}{12} = \frac{3}{6} = \frac{1}{2}$

So, $\frac{3}{6} = \frac{6}{12}$

Therefore, since both fractions are equivalent we can say that the same amount of each pie was eaten.

EXERCISE 159 Date .....

1. Mensa has two cakes that are the same size. The first cake is chocolate, which he cut into 12 equal parts. The second cake is marble, which he cuts into 6 equal parts. His family eats 5 slices of chocolate cake and 3 slices of marble cake. Do they eat more chocolate cake or marble cake?

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2. Joachim bakes two pans of bread that are the same size. One pan has nuts in it and the other pan does not. He cuts the pan with nuts into 8 equal pieces. He cuts the pan without nuts into 16 equal pieces. His friends eat 2 slices of bread with nuts and 3 slices of bread without nuts. Did they eat more of the bread with nuts or without nuts?

3. By which number should we divide the numerator and the denominator of the fraction $\frac{32}{8}$ to make it equivalent to $\frac{8}{2}$?

4. There are 12 fishes in an aquarium and 5 of them are blue in colour. Write two equivalent fractions to represent the blue fishes.

5. Akua bought a bouquet of flowers containing 12 flowers in all. It has 7 roses. Write two equivalent fractions to represent the roses.

Let us use the following examples to review basic operations on fractions involving addition, subtraction, multiplication and division.

Example 550

Find $3\frac{2}{5} + 2\frac{1}{3}$

Solution

$$3\tfrac{2}{5} + 2\tfrac{1}{3} = (3+2) + \left(\tfrac{2}{5} + \tfrac{1}{3}\right)$$
$$= 5 + \left(\tfrac{2\times 3 + 1\times 5}{15}\right)$$
$$= 5 + \left(\tfrac{6+5}{15}\right)$$
$$= 5 + \left(\tfrac{11}{15}\right)$$
$$= 5\tfrac{11}{15}$$

Example 551

Find $4\frac{1}{8} - \frac{3}{4}$

Solution

$$4\tfrac{1}{8} - \tfrac{3}{4} = 4 + \left(\tfrac{1}{8} - \tfrac{3}{4}\right)$$
$$= 4 + \left(\tfrac{3\times 1 - 6\times 3}{24}\right)$$
$$= 4 + \left(\tfrac{3-18}{24}\right)$$
$$= 4 + \left(\tfrac{-15}{24}\right)$$
$$= 3 + 1 + \left(\tfrac{-5}{8}\right)$$
$$= 3 + \tfrac{8}{8} - \tfrac{5}{8}$$
$$= 3 + \tfrac{3}{8}$$
$$= 3\tfrac{3}{8}$$

Example 552

Find $3\frac{3}{5} \times 2\frac{1}{2}$

Solution

$$3\tfrac{3}{5} \times 2\tfrac{1}{2} = \tfrac{3\times 5 + 3}{5} \times \tfrac{2\times 2 + 1}{2}$$
$$= \tfrac{18}{5} \times \tfrac{5}{2}$$
$$= \tfrac{9}{1}$$
$$= 9$$

Example 553

Find $2\frac{1}{8} + \frac{1}{4}$

Solution

$$2\tfrac{1}{8} + \tfrac{1}{4} = \tfrac{2\times 8 + 1}{8} + \tfrac{1}{4}$$
$$= \tfrac{17}{8} + \tfrac{1}{4}$$
$$= \tfrac{17}{8} \times \tfrac{4}{1}$$
$$= \tfrac{17}{2}$$
$$= 8\tfrac{1}{2}$$

BEST BRAIN

EXERCISE 160 Date

Find each of the following.

(a) $3\frac{2}{3} + 1\frac{1}{2}$ (b) $\frac{7}{9} + 4\frac{1}{6}$

(c) $5\frac{1}{4} - 1\frac{2}{5}$ (d) $7\frac{3}{8} - \frac{1}{2}$

(e) $6\frac{1}{3} \times 3\frac{3}{10}$ (f) $\frac{3}{11} \times 3\frac{2}{3}$

(g) $5\frac{1}{4} \div 2\frac{1}{3}$ (h) $3\frac{3}{7} \div 2\frac{5}{14}$

Word problems involving mixed fractions

Example 554

Linda walked $2\frac{1}{3}$ miles on the first day and $3\frac{2}{5}$ miles on the next day. How many miles did she walk in all?

Solution

The total no. of miles she walked is

$= 2\frac{1}{3} + 3\frac{2}{5}$

In the mixed fractions, we have the denominators 3 and 5.

LCM of (3,5) = 15.

To simplify the above expression, we have to make the denominators of both mixed fractions to be 15. Then, we have

$2\frac{1}{3} + 3\frac{2}{5} = 2\frac{5}{15} + 3\frac{6}{15}$

By regrouping, we have

$= (2 + 3) + (\frac{5}{15} + \frac{6}{15})$

$= 5 + \frac{11}{15}$

$= 5\frac{11}{15}$

Example 555

Mr Anto has $3\frac{3}{4}$ acres of land. He gave $1\frac{1}{4}$ acres of land to his friend. How many acres of land does Mr Anto have now?

Solution

Number of acres of land that Mr Anto has

$= 3\frac{3}{4} - 1\frac{1}{4}$

In the mixed fractions, we have the denominators 3 and 4.

LCM of (3,4) = 12.

To simplify the above expression, we have to make the denominators of both mixed fractions to be 12.

Then, we have $3\frac{3}{4} - 1\frac{1}{4} = 3\frac{9}{12} - 1\frac{3}{12}$

By regrouping, we have

$= (3 - 1) + (\frac{9}{12} - \frac{3}{12})$

$= 2 + \frac{6}{12}$

$= 2\frac{6}{12}$

Now, Mr Anto has $2\frac{6}{12}$ acres of land.

Example 556

A bag can hold $1\frac{1}{2}$ pounds of flour. If Mina has $7\frac{1}{2}$ pounds of flour, then how many bags of flour can Mina make?

Solution

No. of bags = Total no. of lbs./No of lbs per bag

Because, we use division, we have to convert the given mixed numbers into improper fractions

Total no. of pounds of flour is $= 7\frac{1}{2} = \frac{15}{2}$

No. of pounds per bag is $= 1\frac{1}{2} = \frac{3}{2}$

Then, we have

Number of bags $= (\frac{15}{2} \div \frac{3}{2})$

$= (\frac{15}{2}) \times (\frac{2}{3})$

$= \frac{30}{6} = 5$

So, the number of bags that Mina can make is 5.

Example 557

Amy has $3\frac{1}{4}$ bottles in her refrigerator. She used $\frac{1}{4}$ bottle in the morning and $1\frac{1}{5}$ bottle in the afternoon. How many bottles of milk does Amy have left over?

Solution

No. of bottles of milk used is $= \frac{1}{4} + 1\frac{1}{5}$

In the mixed fractions, we have the denominators 5 and 4.

L.C.M of (5,4) = 20.

To simplify the above expression, we have to make the denominators of both mixed fractions to be 20.

Then, we have $\frac{1}{4} + 1\frac{1}{5} = \frac{5}{20} + 1\frac{4}{20}$

By regrouping, we have

$= 1 + (\frac{5}{20} + \frac{4}{20})$

$= 1 + \frac{9}{20} = 1\frac{9}{20}$

So, no. of bottles of milk used is $1\frac{9}{20}$

No. of bottles remaining is

$= 3\frac{1}{4} - 1\frac{9}{20}$

(Numerator of the first fraction is smaller than the second. In subtraction of mixed fractions, always the numerator of the first fraction should be greater)

Then, we have

$(3 + \frac{10}{20}) - 1\frac{9}{20} = (2 + 1 + \frac{10}{20}) - 1\frac{9}{20}$

$= (2 + \frac{20}{20} + \frac{10}{20}) - 1\frac{9}{20} = (2 + \frac{30}{20}) - 1\frac{9}{20}$

By regrouping, we have

$= (2 - 1) + (\frac{30}{20} - \frac{9}{20})$

$= 1 + \frac{21}{20} = 1\frac{21}{20}$

So, $1\frac{21}{20}$ bottles of milk Amy has left over.

EXERCISE 161 Date

1. A trader prepared $21\frac{1}{2}$ litres of sobolo. At the end of the day, he had $2\frac{3}{8}$ litres left over. How many litres of sobolo was sold by the Trader?

2. A tank has $82\frac{2}{3}$ litres of water. $24\frac{5}{6}$ litres of water were used and the tank was filled with another $18\frac{3}{4}$ litres. What is the final volume of the water in the tank?

3. James and John went fishing. James caught $3\frac{1}{4}$ kg of fish while John caught $2\frac{1}{5}$ kg of fish. What is the total weight of the fish they caught?

4. In the first hockey games of the year, Tetteh played $1\frac{1}{3}$ periods and $1\frac{1}{4}$ periods. How many periods in all did he play?

5. Akosua added $3\frac{1}{3}$ cups of walnuts to a batch of trail mix. Later she added $1\frac{1}{3}$ cups of almonds. How many cups of nuts did Akosua put in the trail mix in all?

B8.1.3.1.2 Perform addition, subtraction, multiplication and division of integers using BODMAS or PEDMAS

When adding, subtracting, multiplying or dividing integers that involve more than one operation, we use BODMAS or PEDMAS to solve the problem. Work out "bracket", then "of" followed by "division", "multiplication", "addition", and finally workout the "subtraction" This order of operation is referred to as BODMAS. For PEDMAS, work "parenthesis", followed by "exponents", "division", "multiplication", "addition" and then "subtraction"

Example 558

Evaluate $-5-(7-3)-(9-2^3)$

Solution

$-5-(7-3)-(9-2^3)$ (work exponent)
$-5-(7-3)-(9-16)$ (work numbers in bracket)
$-5-(4)-(-7)$ (remove bracket)
$-5-4+7$ (work addition)
$-5+3$
-2
So $-5-(7-3)-(9-2^3)=-2$

Example 559

Evaluate $15\div3+(4\times11)\times5-180$

Solution

$15\div3+(4\times11)\times5-180$ (work numbers in bracket)
$15\div3+44\times5-180$ (work division)
$5+44\times5-180$ (work multiplication)
$5+220-180$ (work addition)
$225-180$ (work subtraction)
45
So $15\div3+(4\times11)\times5-180=45$

Example 560

Evaluate $5^1+25+43-3^1\times4^3+8+(37-14)$

Solution

$5^1+25+43-3^1\times4^3+8+(37-14)$ (work exponent)
$125+25+43-9\times64+8+(37-14)$ (work bracket)
$125+25+43-9\times64+8+23$ (workout division)
$5+43-9\times8+23$ (workout multiplication)
$5+43-72+23$ (work addition)
$48-49$ (workout subtraction)
-1
So $5^1+25+43-3^1\times4^3+8+(37-14)=-1$

Example 561

Evaluate $\frac{1}{2}$ of $\{9^2+(48-27)\div(-28\div4)\}+5^1$

Solution

$\frac{1}{2}$ of $\{9^2+(48-27)\div(-28\div4)\}+5^1$ (workout exponent)
$\frac{1}{2}$ of $\{81+(48-27)\div(-28\div4)\}+25$ (workout bracket)
$\frac{1}{2}$ of $\{81+21\div-7\}+25$ (workout division)
$\frac{1}{2}$ of $\{81-3\}+25$ (workout parenthesis)
$\frac{1}{2}$ of $78+25$ (workout the "of")
$\frac{1}{2}\times78^{39}+25$
$39+25$
64
So $\frac{1}{2}$ of $\{9^2+(48-27)\div(-28\div4)\}+5^1=64$

Example 562

A quiz has 20 questions. Correct answers get +3 and incorrect answers -1 A student answered 5 questions incorrectly. How many points did the student score?

Solution

To solve this question, we will be using the concept of rules for the multiplication of integers.
If 1 answer is correct, 3 points are awarded. Thus, for 15 correct answers, the points acquired will be $15\times3=45$
If 1 answer is incorrect, -1 point is given Thus, for 5 incorrect answers, points will be $(5\times-1)=-5$.
Subsequently, the points scored by the student will be $45-5=40$.
Therefore, the final score is 40 points

Example 563

Ahansewaa bought 4 bananas for 50 pesewas each and 1 apple for 80 pesewas. Write a numerical expression to represent this situation and then find the total cost.

Solution

$4\times50+80=200+80$
$=280$ pesewas or 2.80 cedis
The total cost is GH₵2.80

Example 564

Martha pays 20 cedis for materials to make earrings. She makes 10 earrings and sells 7 for 5 cedis and 3 for 2 cedis. Write a numerical expression to represent this situation and then find Martha's profit.

Solution

Cost of materials for 10 earrings = GH₵ 20
Selling price $=(7\times5)+(3\times2)=35+6=41$
Profit $=41-20=$ GH₵ 21

EXERCISE 162 Date

Evaluate the following.

(a) $39-(14-26)-3^2+4$
(b) $10^1\div20-(4^1-5^2)+18$
(c) $18\times7+9^2\div27-(73-92)$
(d) $12^1\div45-53+18\div9$
(e) $40\div8\times(11-9)+15-4^2$
(f) $\frac{3}{4}$ of $\{13+(22-10)\div(-24\div2)\}\ 2^2$

Real-life problems that involve fractions with a combination of operations

Let's take a look at some real-life problems that involve fractions with a combination of operations. We will use our knowledge of BODMAS or PEDMAS to solve them.

Example 565

Agyiewaa planned to interview some applicants for a position in her office. If she scheduled $\frac{1}{4}$ of an hour to meet each of them, how much time did she schedule for all 6 applicants?

Solution

$\frac{1}{4}\times6=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$
She scheduled $1\frac{1}{2}$ hours to meet the 6 applicants.

Example 566

Instead of $\frac{1}{4}$ of an hour, Agyiewaa spent $\frac{1}{3}$ of an hour to meet each applicant. How much more time did she spend meeting all the applicants?

Solution

$\left(\frac{1}{3}-\frac{1}{4}\right)\times6=\frac{1}{2}$
She spent $\frac{1}{2}$ an hour more than the scheduled time.

Example 567

After the interview, Agyiewaa decided to hire two of the applicants and reject the others. Agyiewaa's assistant called back the applicants to tell them the result. She spent $\frac{1}{30}$ of an hour calling each of the rejected applicants and $\frac{1}{3}$ of an hour calling the hired applicants. How much time did the assistant spend making these calls?

Solution

$\frac{1}{30} \times 4 + \frac{1}{3} \times 2$

$\frac{2}{15} + \frac{2}{3}$ (LCM = 15)

$\frac{2}{15} + \frac{10}{15} = \frac{12}{15} = \frac{4}{5}$

The assistant spent $\frac{4}{5}$ hours to make these calls.

EXERCISE 163 Date

1. Peter withdrew GH¢1000 from his bank account today. He used GH¢500 to fix his car. Then, he divided the remaining into 5 equal parts and gave away 4 parts and kept 1 part for himself. Finally, he took his wife to the restaurant and spent GH¢60 on meals. Write a numerical expression to represent this situation and then find how much money Peter has now.

2. Ewuramaa bought 2 loaves of bread for GH¢10.00 each and 3 chewing gums for GH¢1.20 each.
   Write a numerical expression to represent this situation and then find the cost.

3. John bought 3 pants for 25 cedis each and paid with a one-hundred-cedi note. Write a numerical expression to represent this situation and then find how much money John gets back from the cashier.

4. Yamoah had GH¢35.00 and withdrew GH¢200.00 from his bank account. He bought a pair of trousers for GH¢34.00, Two (2) shirts for GH¢16.00 each, and 2 pairs of shoes for GH¢24.00 each. Give a final expression, and determine how much money Yamoah had at the end of the shopping day.

EXERCISE 164 Date

1. Kojo is printing some materials for a meeting. If the printing of the first copy costs $1\frac{1}{2}$ dollars and other copies cost $\frac{4}{5}$ of a dollar each to print, how much will it cost if he is making 13 copies?

2. Kojo decides to type up some documents while waiting for the meeting to start. He can type 2 pages every $\frac{1}{8}$ hour. If the meeting started $\frac{3}{4}$ hour later than the scheduled time, how many pages can he type before the meeting starts?

3. Kojo is responsible to write the minutes for the meeting. During the meeting, he finished writing $\frac{1}{6}$ of the minutes. Before getting off work, he finished another $\frac{3}{8}$ of the minutes. How much of the minutes does Kojo still need to work on?

B8 1.3.1.3 Solve real life problems involving basic operations on fractions

Now that you understand the concept of gradient and can apply it in determining the equation of a straight line, let us take a look at some real-life problems that make use of the concept of gradient. As you have already learnt, "Gradient is the rate of change". It can therefore be described as a "unit rate" or "common difference".
Let's take a look at these examples.

Example 568

Kwame has five toy cars. He plans to buy two new ones each month.
a) Identify the rate of change.
b) Write an equation for the problem.

Solution

a) Now, to describe the rate of change (gradient), you need to look for key words like each, every, or per. This is your rate of change.
Hence, in this example, the rate of change (gradient) is 2. This is because in the problem, Kwame indicates that he will buy 2 toy cars each month. So we can use the rate of change to tell how many toy cars he will have in, say 3 years' time.

b) The equation of a straight line in the intercept form is given as $y = mx + c$
Where: m = rate of change (slope) and c is the intercept
So, to write the equation, we need m and c
From the problem, $m = 2$ and $c = 5$
Therefore, the equation will be $y = 2x + 5$
Now the x stands for the months and the y stands for the total number of cars.
So, for instance, at the end of the 6th month Kwame will have $y = 2(6) + 5 = 17$
Thus, 17 cars.

Example 569

The initial fee to have a website set up using a server is GH¢48. It costs GH¢44 per month to maintain the website.
a) Identify the rate of change.
b) Write an equation for the problem.

Solution

a) The rate of change is GH¢44.
b) From the problem, $m = 44$ and $c = 48$
Therefore, the equation is $y = 44x + 48$

Example 570

A recording studio charges musicians an initial fee of GH¢50 to record an album. Studio time costs an additional GH¢35 per hour.
a) Write an equation that gives the total cost of an album as a function of studio time (in hours).
b) Find the total cost of recording an album that takes 10 hours of studio time.

Solution

a) From the given problem, $m = 35$ and $c = 50$
Therefore, the equation for the problem is $y = 35x + 50$
b) The total cost of recording for 10 hours is $y = 35(10) + 50 = 350 + 50 = 400$
Therefore, the total cost of recording an album that takes 10 hours of studio time is GH¢ 400.00.

Example 571

1. A dance academy charges GH¢20 to use the facility and GH¢25 per hour of instruction.
   a) Write an equation that gives the total cost to learn dance at the academy as a function of hours of instruction.

b) Find the total cost of 2 hours of dance instruction.

Solution

a. From the given problem, $m = 25$ and $c = 20$
Therefore, the equation for the problem is $y = 25x + 20$

b. The total cost of 2 hours of dance instruction is $y = 25(2) + 20 = 50 + 20 = 70$
Therefore, the total cost of 2 hours of dance instruction is GH¢70.00.

Example 572

Suppose you receive GH¢100 for a birthday present, and you deposit it in a money box. Then each week thereafter, you add GH¢5 to the amount. The amount in the account is a function of the number of weeks that have passed.

a) Find an equation for the amount y you have after x weeks.

b) Use your equation to find when you will have GH¢310 in the account.

Solution

a) $m = 5$ and $c = 100$
Therefore, the equation is $y = 5x + 100$

b) $y = 310$ $\quad 310 = 5x + 100$
$310 - 100 = 5x \quad 210 = 5x$
$\frac{5x}{5} = \frac{210}{5} \quad x = 42$

Therefore, it will take 42 weeks to have GH¢310 in the account.

Example 573

Your gym membership costs GH¢33 per month after an initial membership fee. You paid a total of GH¢228 after 6 months.

a) Write an equation that gives the total cost as a function of the length of your gym membership (in months).

b) Find the total cost after 9 months.

Solution

a) The equation that give the total cost is $y = 33x + c$

b) To find the total cost after 9 months, we first need to find the value of c
Given $x = 6$ and $y = 228$
$228 = 33(6) + c$
$228 = 198 + c$
$c = 228 - 198 = 30$
Now, the total cost after 9 months
$y = 33(9) + 30 \qquad y = 297 + 30 = 327$
The total cost after 9 months = GH¢327.00

Example 574

Mr Anderson gave learners in his class 20 questions to solve. Abiba scored 12 out of the 20 questions. What percentage did she score in the test?

Solution

Fraction of test scored $= \frac{12^{3}}{20_{5}}$

$= \frac{3}{5}$

Percentage $= \frac{3}{5} \times 100^{20} = 60\%$

Example 575

By how much is the sum of $6\frac{1}{4}$ and $3\frac{1}{3}$ greater than 9?

Solution

Sum of $6\frac{1}{4}$ and $3\frac{1}{3} = 6\frac{1}{4} + 3\frac{1}{3}$

$= (6 + 3) + \left(\frac{1}{4} + \frac{1}{3}\right)$

$= 9 + \frac{3 \times 1 + 4 \times 1}{12}$

$= 9 + \frac{3 + 4}{12}$

$= 9\frac{7}{12}$

$9\frac{7}{12} - \frac{9}{1} = \frac{9 \times 12 + 7}{12} - \frac{9}{1}$

$= \frac{108 + 7}{12} - \frac{9}{1}$

$= \frac{115}{12} - \frac{9}{1}$

$= \frac{1 \times 115 - 12 \times 9}{12}$

$= \frac{115 - 108}{12}$

$= \frac{7}{12}$

Therefore the sum of $6\frac{1}{4}$ and $3\frac{1}{3}$ is greater than 9 by $\frac{7}{12}$.

Example 576

Joyce went to school with GH¢20.00. She spent $\frac{3}{5}$ of the money on food.

Find a) fraction of the amount left;
b) the amount spent on food.

Solution

a) Total fraction of money = 1
Fraction of money spent on food + fraction of money left = 1
$\frac{3}{5}$ + fraction of money left = 1
Fraction of money left $= 1 - \frac{3}{5}$

$= \frac{5}{5} - \frac{3}{5}$

$= \frac{5 - 3}{5}$

$= \frac{2}{5}$

b) The amount spent on food $= \frac{3}{5} \times 20^{4}$

$= 3 \times 4$

$= GH¢12.00$

EXERCISE 165 Date

1. A building supply company sells gravel for a fixed delivery fee of GH¢50 per order plus GH¢75 per cubic yard. Write an equation that can be used to find the total cost, c, for gravel, if x represents the number of cubic yards.

2. A moving truck rental costs a flat fee of GH¢30 per day, plus a charge of GH¢0.35 per mile driven. Write an equation that can be used to determine c, the total cost, and x represents the number of miles driven.

3. Paul and Jamie are making necklaces to sell at a craft fair. The tools and other start-up materials cost GH¢12.50. There is an additional cost of GH¢3.25 per necklace. Write an equation that can be used to find the total cost, c, of making a certain number of necklaces, n.

4. The library fine in dollars, F, for an overdue book is GH¢0.25 plus GH¢0.10 per day for each day the book is overdue. Write an equation that can be used to find the total cost, c, and x to represent each day a book is overdue.

5. Mr Mills earns GH¢75 a day plus GH¢35 for each car he sells. If c is his total earnings and x is the number of cars he sold, what is the equation?

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6. A moving truck rental costs a flat fee of GH¢35 per day, plus a charge of GH¢0.25 per mile driven. Write an equation in slope-intercept form.

7. In Bicycle racing, racers purchase a one-year membership to a track. They also pay a fee for each race at that track. One racer paid a total of GH¢125 after 5 races. A second racer paid a total of GH¢170 after 8 races.
   a) How much does the track membership cost?
   b) What is the fee per race?

8. The cost of a school banquet is GH¢95.00 plus GH¢15.00 for each person attending.
   a) Write an equation that gives total cost as a function of the number of people attending.
   b) What is the cost for 77 people?

9. Four years after a hedge maple tree was planted, its height was 9 feet. Eight years after it was planted, the hedge maple tree's height was 12 feet.
   a) What is the growth rate of the hedge maple per year?
   b) What was its height when it was planted?

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BB.1.3.2.1 Perform addition, subtraction, multiplication and division of fractions using BODMAS or PEDMAS

In this section, we shall use the order of operations as outlined in the last two sections to solve problems on fractions.

Example 577

Simplify $\frac{2}{3} + \frac{4}{5} \div \frac{8}{15} - \frac{2}{3}$

Solution

$\frac{2}{3} + \frac{4}{5} \div \frac{8}{15} - \frac{2}{3}$ (workout division)

$\frac{2}{3} + \frac{4}{5} \times \frac{15}{8} - \frac{2}{3}$ (workout multiplication)

$\frac{2}{3} + \frac{3}{2} - \frac{2}{3}$

$\frac{2}{3}$

Example 578

Simplify $28\frac{1}{2} \div \left(2\frac{7}{15} - 1\frac{1}{5}\right)$

Solution

$28\frac{1}{2} \div \left(2\frac{7}{15} - 1\frac{1}{5}\right)$ (workout bracket)

$\frac{28 \times 2 + 1}{2} \div \left((2-1) + \left(\frac{7}{15} - \frac{1}{5}\right)\right)$

$\frac{57}{2} \div \left(1 + \left(\frac{1 \times 7 - 3 \times 1}{15}\right)\right)$

$\frac{57}{2} \div \left(1 + \frac{7-3}{15}\right)$

$\frac{57}{2} \div \left(1 + \frac{4}{15}\right)$

$\frac{57}{2} \div 1\frac{4}{15}$

$\frac{57}{2} \div \frac{19}{15} \Rightarrow \frac{57}{2} \times \frac{15}{19}$

$\Rightarrow 22\frac{1}{2}$

Example 579

Simplify $\left(2\frac{3}{4} - 1\frac{1}{6}\right) \div 1\frac{1}{12} \times \left(\frac{2}{3} - \frac{1}{2}\right)$

Solution

$\left(2\frac{3}{4} - 1\frac{1}{6}\right) \div 1\frac{1}{12} \times \left(\frac{2}{3} - \frac{1}{2}\right)$ (workout bracket)

$\left((2-1) + \left(\frac{3}{4} - \frac{1}{6}\right)\right) \div \frac{1 \times 12 + 1}{12} \times \left(\frac{2 \times 2 - 1 \times 3}{6}\right)$

$\left(1 + \left(\frac{3 \times 3 - 1 \times 2}{12}\right)\right) \div \frac{13}{12} \times \left(\frac{4-3}{6}\right)$

$\left(1 + \frac{9-2}{12}\right) \div \frac{13}{12} \times \frac{1}{6}$

$1\frac{7}{12} \div \frac{13}{12} \times \frac{1}{6}$

$\frac{19}{12} \times \frac{12}{13} \times \frac{1}{6}$

$\frac{19 \times 1}{13 \times 6}$

$\frac{19}{78}$

EXERCISE 166 Date

Simplify each of the following.

(a) $\frac{5}{6} \div 3\frac{1}{3} + \frac{1}{2}$ (b) $1\frac{1}{2} - \frac{2}{5} \div 2\frac{2}{5}$

(c) $\frac{3}{5} + \frac{2}{3} \div 1\frac{1}{3} - \frac{3}{4}$ (d) $9\frac{1}{6} \div \left(2\frac{5}{6} - 1\frac{1}{2}\right)$

(e) $\left(\frac{1}{10} \text{ of } 3\frac{1}{8}\right) \div \left(4\frac{1}{2} - 1\frac{3}{4}\right)$

(f) $1\frac{1}{6} \div \frac{4}{9} \text{ of } \left(\frac{2}{3} + \frac{1}{4}\right)$

BEST BRAIN

Example 580

The total population of learners in Fiapre D/A Basic School is 250. $\frac{2}{5}$ of the population are boys and the rest are girls. Find
a) fraction of girls in the school;
b) the number of girls in the school.

Solution

a) Total fraction of learners = 1

Fraction of boys + fraction of girls = 1

$\frac{3}{5}$ + fraction of girls = 1

Fraction of girls in the school = $1 - \frac{2}{5}$

$= \frac{5}{5} - \frac{2}{5}$

$= \frac{5-2}{5}$

$= \frac{3}{5}$

b) Number of girls in the school = $\frac{3}{5} \times 250^{50}$

$= 3 \times 50$

$= 150$

EXERCISE 167 Date

1. The sides of a triangle are $2\frac{1}{2}$ m, $3\frac{1}{3}$ m and 1 m. Find the perimeter of the triangle.
2. Antoinette scored 24 out of 40 questions in a test. Find the percentage of her score in the test.
3. In a school, $\frac{7}{10}$ of the learners comes to school on foot and the rest come to school by car. Find
   a) fraction of learners who come to school by car;
   b) percentage of learners who come to school by car.
4. A woman bought 27 litres of palm nut oil. She sold $\frac{5}{9}$ in the first week. Find the
   a) fraction of oil left;
   b) number of litres of oil sold in the first week;
   c) number of litres of oil left.
5. Alhasan went to school with GH¢30.00. She spent $\frac{1}{6}$ of the money on books. Find the
   a) fraction of the amount left;
   b) amount spent on books;
   c) amount of money left.
6. The total number of workers in a company is 408. $\frac{3}{8}$ of the workers are female and the rest are male. Find
   a) fraction of males in the company;
   b) the number of males in the company;
7. A school had 645 furniture. At the end of the year, $\frac{4}{15}$ of the furniture had broken down. Calculate the
   a) fraction of furniture that have not broken down;
   b) number of furniture that have not broken down.

STRAND 1.4 RATIO AND PROPORTION

B8.1.4.1.1 Use ratio reasoning to convert measurement units

In Basic 4, 5 and 6, we learnt how to convert unit measurement from lower to higher and vice versa. In this section, we shall use our knowledge on ratio from Basic 7 to convert measurement units.

Example 581

Convert 438 mm to metres

Solution

We know that

1,000 mm = 1m or 1 mm = $\frac{1}{1000}$ m

438 mm = 438×1 mm

$= 438 \times \frac{1}{1000}$ m

$= \frac{438}{1000}$ m

$= 0.438$ m

Example 582

Convert 0.723 kg to grams

Solution

We know that

1,000g = 1 kg or 1 g = $\frac{1}{1000}$ kg

0.723kg = 0.723×1 kg

$= 0.723 \times 1000$g

$= \frac{723}{1000} \times 1000$ g

$= 723$ g

Example 583

Ato can read 7 pages of a book in 21 minutes. Amina can also read 10 pages of the same book in 25 minutes. Which of them reads faster?

Solution

Rate at which Ato can read

$= \frac{21^{3}}{7} = 3$ minutes for each page

Rate at which Amina can read

$= \frac{25}{10} = 2.5$ minutes for each page

Therefore Amina reads faster than Ato.

EXERCISE 168 Date

1. Convert 316 mm to metres
2. Convert 716 m to kilometres.
3. Convert 86 cm to metres.
4. Convert 0.766 litres to millilitres.
5. Fiifi can weed 0.6 acres in 4 hours. Sumailla can also weed 0.5 acres of the same land in 3 hours. Which of them weeds faster?
6. Abieku can work 15 questions in 1 hour 15 minutes. Nortey can also work 20 questions of the same difficulty in 2 hours. Which of them can work faster?

B8.1.4.1.2 Solve unit rate problems

A rate is a comparison or ratio of two quantities with different units. When we compare a quantity to a unit of another quantity, we call it unit rate. In this section, we shall use our knowledge on ratio to solve unit rate problems involving unit pricing and constant speed.

Example 584

Afrifa walks 15 miles in 3 hours. How many miles does he walk per hour?

Solution

$$\text{Distance walked per hour} = \frac{15 \text{ miles}}{3 \text{ hours}}$$
$$= \frac{5 \text{ miles}}{\text{hours}}$$
$$= 5 \text{ miles per hour}$$

Unit rate that deals with distance and time is called speed.

We can therefore say that Afrifa's speed is 5 miles per hour.

Example 585

A shop sells 4 T-shirts for GH₵128.00. What is the cost per T-shirt?

Solution

$$\text{Cost per T-shirt} = \frac{128}{4} \text{ (rate)}$$
$$= \text{GH₵32.00}$$

Example 586

3 litres of gasoline cost GH₵13.50. Find
(a) the cost of 1 litre of gasoline;
(b) the cost of 13 litres of gasoline.

Solution

(a) $\text{Cost of 1 litre of gasoline} = \frac{13.5}{3}$
$$= \frac{13.5}{3 \times 10}$$
$$= \frac{45}{10}$$
$$= \text{GH₵4.50 per litre}$$

(b) $\text{Cost of 13 litres} = \frac{\text{GH₵4.50}}{1 \text{ litre}}$
$$= \text{GH₵} \frac{45}{10} \times 13$$
$$= \text{GH₵} \frac{585}{10}$$
$$= \text{GH₵ 58.50}$$

Example 587

A car travels 21 km on $\frac{3}{4}$ gallon of diesel. Find the
(i) number of kilometres the car can travel on 1 gallon;
(ii) number of gallons the car can use to travel 252 km.

Solution

(i) $\frac{\text{kilometres}}{\text{gallons}} = \frac{21}{\frac{3}{4}}$
$$= 21 \div \frac{3}{4}$$
$$= 21 \times \frac{4}{3}$$
$$= 28$$

Therefore the car can travel 28 km per gallon.

BEST BRAIN

(ii) The unit rate found in (i) was km per gallon, if we need gallons for a given kilometer we shall reciprocate the results in (i).

$$\frac{\text{gallons}}{\text{kilometres}} = \frac{1}{28}$$

Number of gallons needed to travel 252 km
$$= \frac{1 \text{ gallon}}{28 \text{ km}} \times 252 \text{ km}$$
$$= \frac{1 \times 252}{28} \text{ gallon}$$
$$= 9 \text{ gallons}$$

EXERCISE 169 Date

1. Ampadu travels 32 kilometres in every 2 hours. How many kilometres does he travel in 1 hour?

2. A machine can pump 693 litres of water in $\frac{3}{5}$ hours. Find the amount of litres the machine can pump in
   (i) 1 hour;
   (ii) 5 hours.

3. Mahama pays a company GH₵132.00 every 6 months to host his website. How much does Mahama pay
   (a) per month;
   (b) for 11 months?

4. A machine can print 1,750 sheets in 5 minutes.
   (a) What is the unit rate of the machine in sheets per minutes?
   (b) How many sheets can the machine print in 35 minutes?
   (c) How long will it take the machine to print 6,300 sheets?

5. Kuma can bike 42 km in 3 hours.
   (a) What is the unit rate (speed) of Kuma?
   (b) How many kilometres can he bike in 5 hours 30 minutes?
   (c) How many hours will he take to cover 175 km?

BEST BRAIN

B8.1.4.1.3 Apply the knowledge of speed to draw and interpret distance time graphs

We travel from one place to another by foot, bike, a car, a ship or by aeroplane. The movement of a person from one place to another can be described using a distance time graph. In this section, we shall use our knowledge from the previous section to draw and interpret distance time graph.

Example 588

Ampadu left the house at 8:00 am and walked to his friend's house a distance of 5.4 km. He reached the friend's house at 10:00 am. He stayed in the house for 2 hours and returned to his house. He reached the house at 1:30 pm.

(a) Using a scale of 2 cm to 1 hour on x-axis and 2 cm to 1 km on y-axis, draw a distance time graph for Ampadu's journey.

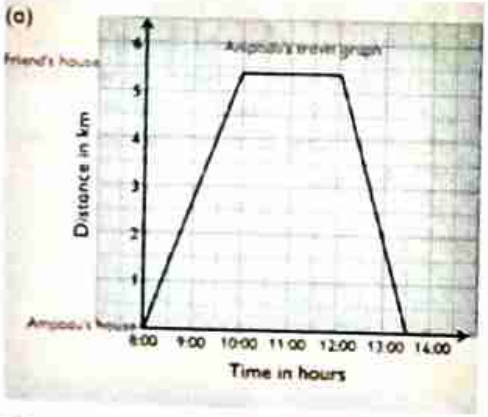
(b) Find Ampadu's speed from his house to the friend's house.

(c) The total distance travelled by Ampadu.

(d) Find Ampadu's return speed.

Solution

(a)



(b) $\text{Speed} = \frac{\text{Distance covered}}{\text{Time}}$

Ampadu's speed from his house to the friend's house $= \frac{5.4 \text{ km}}{2 \text{ hours}}$

$= \frac{54 \text{ km}}{10 \times 2}$

$= 2.7$ km/h

(c) The total distance travelled by Ampadu $= 5.4 \text{ km} + 5.4 \text{ km}$

$= 10.8$ km

(d) He left the friend's house at 12:00 and got home at 13:30, it means that he used 1 hour 30 minutes or 1.5 hours

His return speed $= \frac{5.4 \text{ km}}{1.5 \text{ h}}$

$= \frac{54}{15} = 3.6$ km/h

Example 589

Fosuaa boarded a car from station P at 5:00 am. The car travelled at a speed of 40 km/h for 1 hour 30 minutes. The car stopped at station B for 30 minutes and continued for 1 hour covering a distance of 50 km to reach town K. Fosuaa rested for 1 hour and returned home. She got back to station P at exactly 11:00 am.

(a) Find the distance covered by the car in the first 1 hour 30 minutes.

(b) Draw a distance time graph for Fosuaa, using a scale of 1 hour on x-axis and 2 cm to 20 km on y-axis.

c) Find the total distance covered by Fosuaa.

BEST BRAIN

(d) Find the speed of the car during the return journey.

(e) Find the average speed for the whole journey.

Solution

(a) 1 hour = 60 minutes

1 minute $= \frac{1}{60}$ hours

30 minutes $= \frac{30}{60} = 0.5$ hours

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$40 = \frac{\text{Distance}}{1.5}$ (multiply both sides by 1.5)

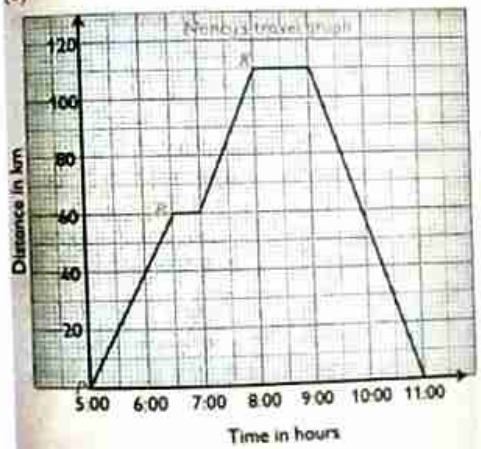
$40 \times 1.5 = \frac{\text{Distance}}{1.5} \times 1.5$

$40 \times \frac{15}{10} = \text{Distance}$

$4 \times 15 = \text{Distance}$

$60 \text{ km} = \text{Distance}$

(b)



(c) Total distance covered $= 60 + 50 + 110$

$= 220$ km

(d) She left town K at 9:00 and got back to station P at 11:00 making 2 h.

Return speed $= \frac{110}{2}$

$= 55$ km/h

(e) Average speed is the total distance for the journey divided by the total time.

Time from P to $B = 1.5$ h, Time from B to $K = 1$ h and Time from K back to $P = 2$ h.

Total time $= 1.5 + 1 + 2 = 4.5$ h

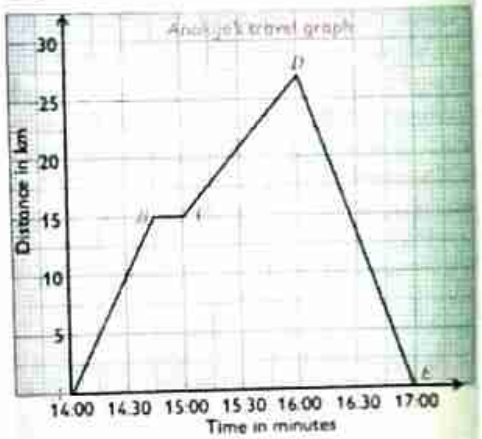
Average speed $= \frac{220}{4.5} = \frac{220 \times 10}{45}$

$= \frac{440}{9}$

$= 48\frac{8}{9}$ km/h

Example 590

The distance time graph below shows the movement of Anokye on a motorbike in a given day.



(a) How long did he stop at B?

(b) How long is the distance from C to D?

(c) Find his speed from A to B.

(d) What is his return speed?

(e) Find the total distance travelled.

BEST BRAIN

Solution

(a) He stopped for 15 minutes

(b) Distance from C to $D = 27 - 15$
$= 12$ km

(c) Time from A to $B = 45$minutes $= \frac{3}{4}$ hour

Speed from A to $B = 15 \div \frac{3}{4}$
$= 15 \times \frac{4}{3}$
$= 20$ km/h

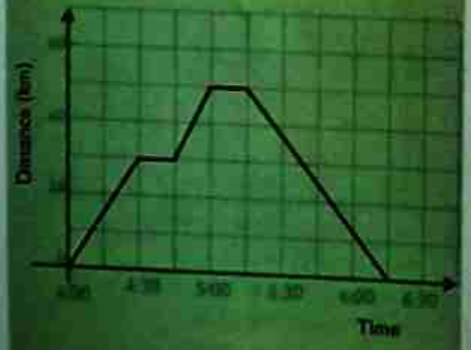
(d) He used 1 hour to move from D to E

Return speed $= \frac{27}{1}$
$= 27$ km/h

(e) Total distance $= 15 + 12 + 27$
$= 54$ km

Example 591

Sefe is going for a drive in his car. The distance-time graph given below describes his full journey. Calculate his total distance travelled during his journey, as well as his average speed between 4:30p.m. and 5:30p.m.



Solution

Sefa traveled 30 km away from his home and then stopped for a while. He again drove 20 km and stopped briefly. He then traveled 50km back home.

So, the total distance traveled by Sefa
$= 30$ km $+ 20$ km $+ 50$ km $= 100$ km.

From the axis, we can observe that two big squares total 30 minutes. Hence, one big square is worth 15 minutes. From 4:30 to 4:45, Sefa is at rest.

So, his speed between 4:30 and 4:45
$= 0/0.25 = 0$ km/h.

To calculate the average speed of Sefa between 4:30 and 5:30, we need to calculate the slope of the graph between 4:45 and 5:00. This period lasted for 15 minutes, which is equivalent to 0.25 hours – this is the "change in x". During this period, he increased his distance from home from 30 km up to 50 km. That means he travelled 20 km in total – this is the "change in y".

So, we get, slope $= 20/0.25 = 80$ km/h.

Hence, the average speed $= (0 + 80)/2 = 40$ km/h

BEST BRAIN

EXERCISE 170 Date

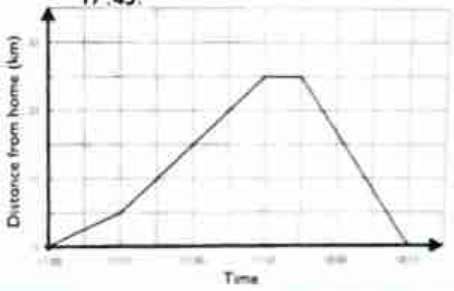
1. Agbo was sent from the house to a shop at 9:00 am. The distance from the house to the shop is 320 m, he got to the shop at 9:20 am, spent 10 minutes at the shop and returned to the house. He arrived to the house at 9:45 am.
   (a) Draw a distance time graph for Agbo's journey, using a scale of 2 cm to 5 minutes on the x-axis and 2 cm to 50 m on y-axis.
   (b) Find Agbo's speed from his house to the shop in metres per minutes.
   (c) The total distance travelled by Abgo.
   (d) Find the return speed of Agbo.

2. One morning, Akata walked to the school at a speed of 20 m/min and got to the school after 40 minutes.
   He immediately returned without resting and arrived home after 25 minutes.
   (a) Find the distance between his house and the school.
   (b) Draw a distance time graph for Akata, using a scale of 2 cm to 10 minutes on x-axis and 2 cm to 100 m on y-axis.
   (c) Find the return speed of Akata.

3. A woman left her home at 7:00 am in her car and arrived at another town A, 20 km away after 20 minutes. She stopped at town A for 10 minutes and continued at a speed of 66 km per hour for 20 minutes in another town B. She returned immediately without stopping at a speed of 72 km per hour till she got home.
   (a) Find the time she used from town B to the house.
   (b) Use a scale of 2 cm to 10 minutes on x-axis and 2 cm to 10 km on y-axis, draw a distance time graph for the woman.
   (c) Find the total distance covered.
   (d) Find the speed from the house to town A.

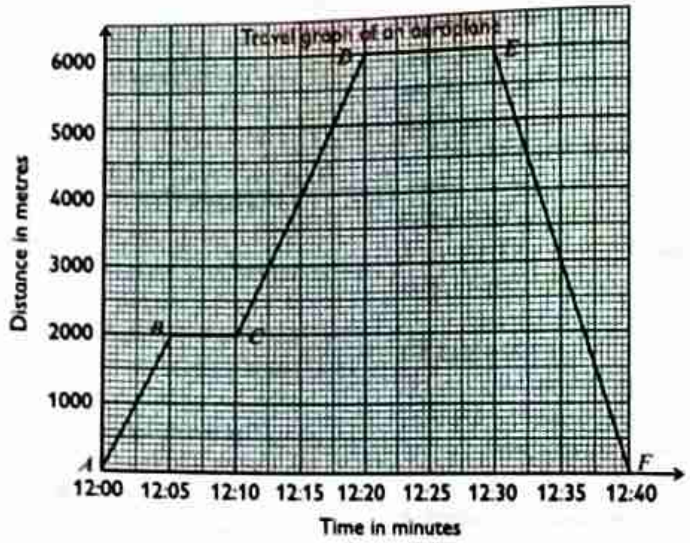
4. Amadu was sent by his parents to send some items to their friend in another town 3.2 km away. Amadu left home at 8:30 am walked for 1 hour and covered a distance of 2 km, he rested for 30 minutes and continued. He spent 30 minutes in the friend's house and returned to his house using 1 hour 24 minutes.
   (a) Draw a distance time graph for Amadu, using a scale of 2 cm to 30 minutes on x-axis and 2 cm to 0.5 km on y-axis.
   (b) How many minutes did Amadu rest?
   (c) Find Amadu's return speed.
   (d) Find the speed of Amadu from his house to the first rest point.

5. Adwoa is going for a bike ride. Below is a distance-time graph that describes her full journey.
   a) How long was she stationary for?
   b) What was the total distance travelled during her journey?
   c) What was her average speed in kilometres per hour between 17 15 and 17 :45?



BEST BRAIN

7. The distance time graph below shows the movement of an aeroplane from town A to town E.



(a) How many minutes did the aeroplane cruise at height 2,000 m?

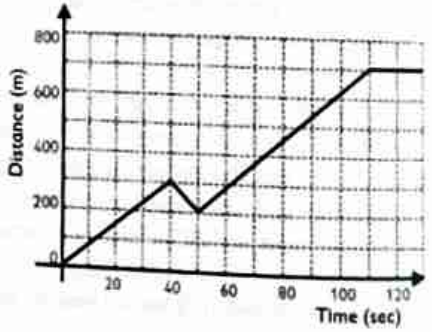
(b) Find the speed of the aeroplane from A to B in metres per minutes.

(c) How many minutes did the aeroplane cruise at height 6,000 m?

(d) What is the speed of the aeroplane from E to F?

(e) Find the average speed of the aeroplane

8. Every morning, Shaibu walks from his home to the bus stop nearby, which is 750 meters away from his home. The graph below shows his journey on one particular day. Calculate his average speed from 0 sec to 40 sec and from 50 sec to 110 sec.



B8.1.4.1.4 Use equivalent ratios to determine proportionality between relationships

In this section, we shall use our knowledge on ratios to determine proportionality or non-proportionality of relationships.

Example 592

Use ratios to determine the proportionality or otherwise on the distance travelled by two friends as shown in the tables.

Ansu's distance with time			
Distance (km)	2	4	6
Time (hr)	1	2	3

Amina's distance with time			
Distance (km)	4.5	6	10
Time (hr)	1.5	2	2.5

Solution

Let us use ratios to determine the equivalence of the distance travelled by the two friends.

Ansu

$\frac{2}{1} = 2$, $\frac{4}{2} = 2$ and $\frac{6}{3} = 2$

It means that $\frac{2}{1} = \frac{4}{2} = \frac{6}{3} = 2$ (equivalent ratios)

Distance covered by Ansu with time is proportional.

Amina

$\frac{4.5}{1.5} = 3$, $\frac{6}{2} = 3$ and $\frac{10}{2.5} = 2$

It means that $\frac{4.5}{1.5} \neq \frac{10}{2.5}$ (not equivalent ratios)

Distance covered by Amina with time is not proportional.

Example 593

The graphs show the growth of three different trees. Describe which of them is proportional and explain why.

Solution

Wawa tree growth

We can see that the line passes through the

origin but there are points outside the line. The growth is not proportional.

Teak tree growth
We can see that the line passes through the origin and all the points lie on it. The growth is proportional.

Onyina tree growth
We can see that the line does not pass through the origin. The growth is not proportional.

EXERCISE 171 Date

1 Use ratios to determine the proportionality or otherwise on the weight of three children as shown in the tables.

(a)

Mohammed			
Weight (kg)	35	40	50
Time (years)	10	12	15

(b)

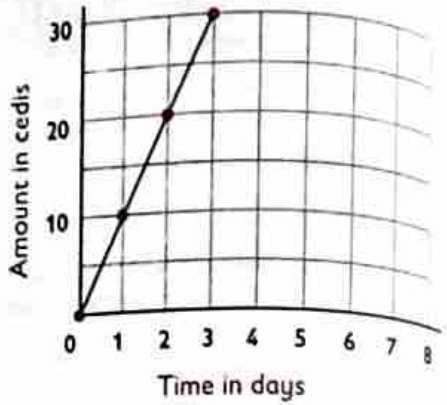
Ackah			
Weight (kg)	20	25	30
Time (years)	4	5	6

(c)

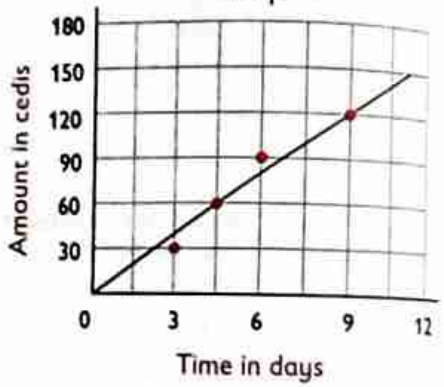
Dede			
Weight (kg)	35	49	64
Time (years)	5	7	8

2 The graphs show the sales made by two different shop owners over a period of time. Describe which of them is proportional and explain why.

Shop A



Shop B



BEST BRAIN

B8 1.4.1.5 Identify the constant of proportionality in tables, graphs and equations

In the last two previous sections, we learnt how to find unit rate. In this section, we shall use the concept of unit rate to find constant of proportionality in tables, graphs or equations.

Example 594

A man walked a distance of $\frac{3}{2}$ km in 30 minutes and $3\frac{3}{4}$ km in 1 hour 15 minutes. Find the constant of proportionality.

Solution

We know that 30 minutes = $\frac{1}{2}$ hour

Constant of proportionality (k) = $\frac{3}{2} \div \frac{1}{2}$

$= \frac{3}{2} \times \frac{2}{1}$

$= 3$

Note that 1 hour 15 minutes = $1\frac{1}{4}$ hours

Or constant of proportionality (k) = $3\frac{3}{4} \div 1\frac{1}{4}$

$= \frac{15}{4} \div \frac{5}{4}$

$= \frac{15}{4} \times \frac{4}{5}$

$= 3$

Example 595

The table shows the price in cedis of mutton in pounds in a meat shop.

Pounds (lb)	2	3	4	5
Amount (cedis)	8	12	16	20

Determine the constant of proportionality.

Solution

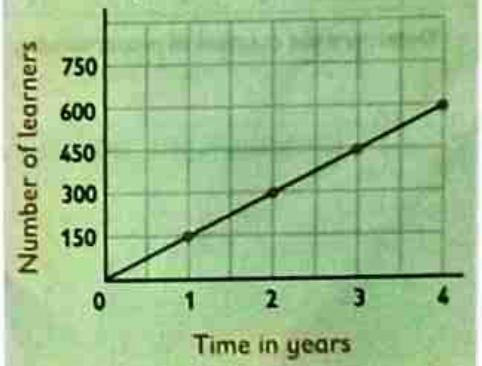
We can find the constant of proportionality by using any one of the ratios.

Constant of proportionality (k) = $\frac{8}{2} = 4$

Or Constant of proportionality (k) = $\frac{20}{5} = 4$

Example 596

The graph shows total number of learners in a school over a period of 4 years.



(a) Create a table using the points on the graph.
(b) Find the constant of proportionality.

Solution

(a)

Time (in years)	1	2	3	4
Number of learners	150	300	450	600

(b)

Constant of proportionality (k) = $\frac{150}{1} = 150$

Or Constant of proportionality (k) = $\frac{450}{3} = 150$

BEST BRAIN

EXERCISE 172 Date

1. Atongo can lay 14 blocks in 10 minutes and 91 blocks in 1 hour 5 minutes.
   Find the constant of proportionality.

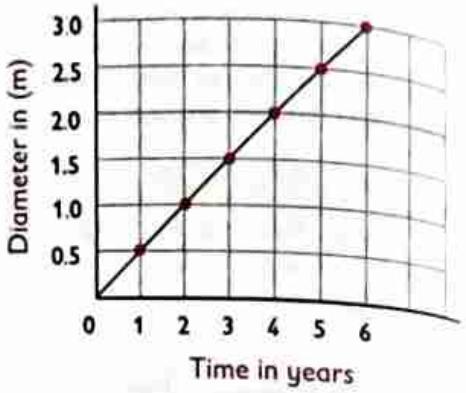
2. A machine can photocopy 124 sheets in 4 minutes and 1,612 sheets in 52 minutes.
   Find the constant of proportionality.

3. The table shows the volume of water a machine can pump in a given time.

Volume (litres)	216	360	504	720
Time (minutes)	3	5	7	10

   Determine the constant of proportionality.

4. The graph shows the growth of a tree in a given forest reserve.



   (a) Create a table using the points on the graph.
   (b) Find the constant of proportionality.

STRAND 2: ALGEBRA

SUB-STRANDS

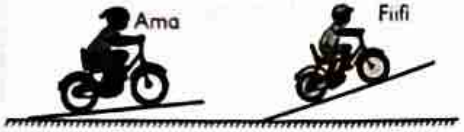
- Patterns and relations
- Algebraic expressions
- Variables and equations

STRAND 2.1

PATTERNS AND RELATIONS

B8.2.1.1.1 Use real life object to explain the concept of gradient

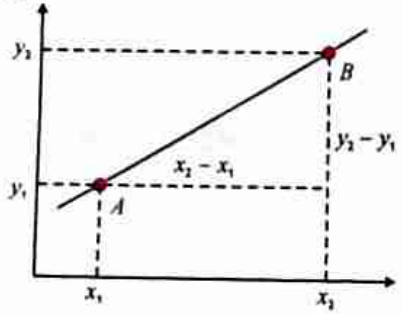
Look at the children climbing a hill on a bicycle as shown.



Which of them can easily climb the hill? We can see that Ama can easily climb the hill because the hill she is climbing is gentle whiles the hill Fiifi is climbing is steeper.

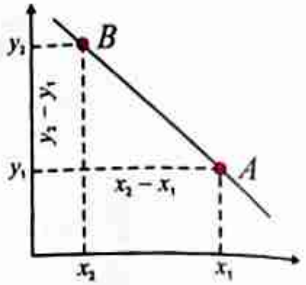
How steep or gentle slope is can be described as gradient.

Consider the two points on the line shown in the diagram below.



AB shows a line up the hill.

$$\text{Gradient of } AB = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



AB shows a line down the hill.

$$\text{Gradient of } AB = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 597

Determine the gradient of the line that passes through the points $A(3, 7)$ and $B(5, 15)$.

Solution

Let $A(3,7) = (x_1, y_1)$ and $B(5, 15) = (x_2, y_2)$

$$\text{Gradient of } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 7}{5 - 3}$$

$$= \frac{8}{2}$$

$$= 4$$

To find gradient of a line through two points, one can use any of the points for (x_1, y_1) and the other for (x_2, y_2).

BEST BRAIN

Example 598

Determine the gradient (slope) of the line that passes through the points $P(-1, 3)$ and $Q(-3, -9)$.

Solution

Let $P(-1, 3) = (x_1, y_1)$ and $Q(-3, -9) = (x_2, y_2)$

$$\text{Gradient of } PQ = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9 - 3}{-3 - (-1)}$$

$$= \frac{-12}{-3 + 1}$$

$$= \frac{-12}{-2}$$

$$= 6$$

Example 599

The slope of the line through the points $C(-4, -1)$ and $D(1, b)$ is 1. Find the value of b.

Solution

Let $C(-4, -1) = (x_1, y_1)$ and $D(1, b) = (x_2, y_2)$

$$\text{Gradient of } CD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b - (-1)}{1 - (-4)}$$

But the gradient of the slope = 1

$$1 = \frac{b + 1}{5} \quad \text{(multiply both sides by 5)}$$

$$1 \times 5 = b + 1$$

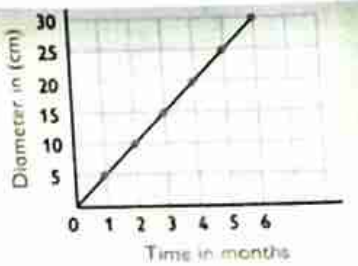
$$5 = b + 1$$

$$5 - 1 = b + 1 - 1$$

$$4 = b$$

Example 600

The graph shows the relation between the diameter of a tree with time.



Find the growth (gradient) of the tree.

Solution

To find the growth (gradient), we shall write down the coordinates of any two points (1, 5) and (3,15)

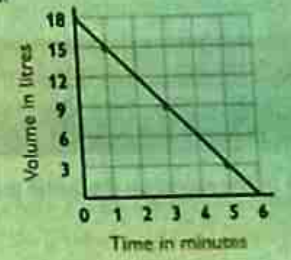
$$\text{Gradient} = \frac{15 - 5}{3 - 1}$$

$$= \frac{10}{2}$$

$$= 5$$

Example 601

The graph shows the volume of water in a tank with time.



Find the rate at which water is decreasing from the tank (gradient).

Solution

To find the gradient, we shall write down the coordinates of any two points (1,15) and (5,3)

$$\text{Gradient} = \frac{3 - 15}{5 - 1}$$

BEST BRAIN

Gradient $= \frac{-12}{4}$

$= -3$

From the preceding two examples, it can be observed that a line that slopes up the hill has a positive gradient and a line that slopes down the hill has a negative gradient.

EXERCISE 173 Date

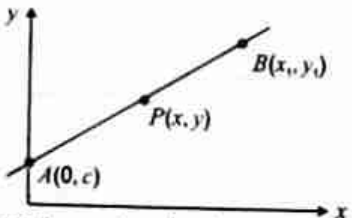
1. Determine the gradient of a line which passes through the points
   (a) $A(2,1)$ and $B(1,6)$
   (b) $M(-3,2)$ and $N(2,12)$
   (c) $P(5,-2)$ and $Q(2,-8)$
   (d) $C(11,-20)$ and $D(8,1)$

2. The slope of the line through the points $X(4, y)$ and $Y(7,3)$ is 2. Find the value of y.

3. The slope of the line through the points $B(1,-5)$ and $C(4,m)$ is 3. Find the value of m.

Using the concept of gradient to establish equation of a line in the form $y = mx + c$

Let us look at the points on the line in the graph below.



There are three points that lie on the line. The gradient of AP and the gradient of AB will be the same since A, P and B all lie on the line.

Let the gradient of $AB = m$.

Gradient of $AP = \frac{y-c}{x-0}$

But gradient of AB = gradient of AP

$m = \frac{y-c}{x}$ (multiply both sides by x)

$mx = y - c$ (add c to both sides)

$y = mx + c$

This equation is called equation of a straight line in slope intercept form.

Note that m is the gradient and c is the y-intercept.

Example 602

Find the gradient and y-intercept from the equation of the line $y = 7x + 3$.

Solution

The line $y = 7x + 3$ is in the form $y = mx + c$

Comparing: gradient $(m) = 7$ and y-intercept $(c) = 3$

Example 603

Find the gradient and y-intercept from the equation of the line $6x - 2y - 4 = 0$.

Solution

We first need to write the equation in the form $y = mx + c$

$6x - 2y - 4 = 0$ (add $2y$ to both sides)

$6x - 2y + 2y - 4 = 2y$

$6x - 4 = 2y$ (divide both sides by the number multiplying y)

$\frac{6x}{2} - \frac{4}{2} = \frac{2y}{2}$

$3x - 2 = y$

Comparing: gradient $(m) = 3$ and y-intercept $(c) = -2$

BEST BRAIN

Example 604

Find the equation of a line with slope 2 and y-intercept -5.

Solution

Equation of the line is $y = 2x - 5$

We can also find the equation by first writing the y-intercept as $(0, -5)$ and choose any point (x, y) on the line.

Gradient $= \frac{y-(-5)}{x-0}$

$2 = \frac{y+5}{x}$

$2x = y + 5$

$y = 2x - 5$

Example 605

Find the equation of a line with gradient $\frac{3}{2}$ and y-intercept 1. Hence find the value of y when $x = -8$

Solution

Equation of the line is $y = \frac{3}{2}x + 1$

We can also find the equation by first writing the y-intercept as $(0, 1)$ and choose any point (x, y) on the line.

Gradient $= \frac{y-1}{x-0}$

$\frac{3}{2} = \frac{y-1}{x}$

$\frac{3}{2}x = y - 1$

$y = \frac{3}{2}x + 1$

When $x = -8$ $y = \frac{3}{2} \times (-8) + 1$

$y = 3 \times (-4) + 1$

$= -12 + 1$

$= -11$

Example 606

Find the equation of a line that passes through the point $(-1,-3)$ and has a slope of $\frac{2}{5}$

Solution

Let (x, y) and $(-1,-3)$ be points on the line.

Gradient $= \frac{y-(-3)}{x-(-1)}$

$\frac{2}{5} = \frac{y+3}{x+1}$ (multiply both sides by 5)

$2 = \frac{5(y+3)}{x+1}$ (multiply both sides by $x + 1$)

$2(x + 1) = 5(y + 3)$ (using distributive property)

$2 \times x + 2 \times 1 = 5 \times y + 5 \times 3$

$2x + 2 = 5y + 15$ (group all to one side)

$2x - 5y + 2 - 15 = 0$

$2x - 5y - 13 = 0$

EXERCISE 174 Date

1. Find the gradient and y-intercept from the equation of the line $y = 3x - 10$.

2. An equation of a straight line $y = \frac{3}{2}x + 5$. Find the gradient and y-intercept.

3. $2y = 6x - 2$ is an equation of a straight line, find the gradient and the y-intercept.

4. An equation of a straight line is $12x + 3y - 6 = 0$. Find the gradient and the y-intercept.

5. Find the equation of a straight line with slope 1 and y-intercept of 2.

BEST BRAIN

6. An equation of a straight line has slope $\frac{3}{5}$ and y-intercept -2. Find the equation of the line. Hence find the value of y when $x = -20$.

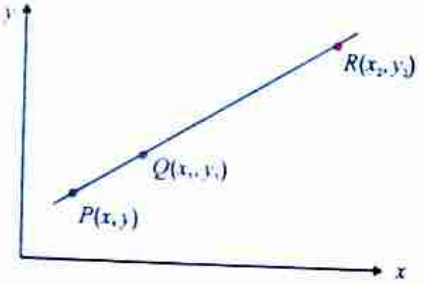
7. Find the equation of a line with gradient $-\frac{4}{7}$ and y-intercept 5. Hence find the value of y when $x = 21$.

8. Find the equation of a line that passes through the point (3, 4) and has a slope of 2.

9. A straight line with slope $\frac{1}{3}$ passes through the point $(-3, 5)$.
Find the equation of the line.

Using the concept of gradient to establish equation of a line through two points

Consider the points P, Q and R on a straight line as shown.



Since P, Q and R all lie on the line, the gradient of PQ, the gradient of PR and the gradient of QR are all the same.

Gradient of $PQ = \frac{y - y_1}{x - x_1}$

Gradient of $QR = \frac{y_2 - y_1}{x_2 - x_1}$

Therefore $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

Example 607

The points $A(2,3)$, $B(5,b)$ and $C(1,-3)$ all lie on a straight line. Find the value of b.

Solution

Gradient of $AB = \frac{b-3}{5-2}$

$= \frac{b-3}{3}$

Gradient of $AC = \frac{-3-3}{1-2}$

$= \frac{-6}{-1}$

$= 6$

But gradient of AB = gradient of AC

$\frac{b-3}{3} = 6$ (multiply both sides by 3)

$b - 3 = 6 \times 3$

$b - 3 = 18$

$b = 18 + 3$

$b = 21$

Example 608

A straight line passes through the points $M(-2,5)$ and $N(1,11)$. Find the equation of the line.

Solution

Let $R(x,y)$ be a point on the line

Gradient of $MN = \frac{11-5}{1-(-2)}$

$= \frac{6}{1+2}$

$= \frac{6}{3} = 2$

Gradient of $MR = \frac{y-5}{x-(-2)}$

BEST BRAIN

Gradient of $MR = \frac{y-5}{x+2}$

But gradient MN = gradient of MR

$2 = \frac{y-5}{x+2}$ (multiply both sides by $x+2$)

$2(x+2) = y - 5$ (using distributive property)

$2 \times x + 2 \times 2 = y - 5$

$2x + 4 = y - 5$

$2x + 4 + 5 = y$

$2x + 9 = y$

Example 609

$C(1,-8)$ and $D(7,-4)$ are points on a straight line. Find the equation of the line.

Solution

Let $M(x,y)$ be a point on the line

Gradient of $CD = \frac{-4-(-8)}{7-1}$

$= \frac{-4+8}{6}$

$= \frac{4}{6} = \frac{2}{3}$

Gradient of $CD = \frac{y-(-8)}{x-2}$

$= \frac{y+8}{x-2}$

$\frac{2}{3} = \frac{y+8}{x-2}$ (multiply both sides by 3)

$2 = \frac{3(y+8)}{x-2}$ (multiply both sides by $x-2$)

$2(x-2) = 3(y+8)$ (using distributive property)

$2 \times x - 2 \times 2 = 3 \times y + 3 \times 8$

$2x - 4 = 3y + 24$ (group all to one side)

$2x - 4 - 3y - 24 = 0$

$2x - 3y - 28 = 0$

EXERCISE 175 Date

1. The points $D(4,1)$, $B(h,7)$ and $C(6,5)$ all lie on a straight line. Find the value of h.

2. $P(7,3)$, $Q(4,k)$ and $R(5,-5)$ are points on a straight line. Find the value of k.

3. A straight line passes through the points $K(9,6)$ and $L(5,-6)$.
Find the equation of the line.

4. Find the equation of the straight line which passes through the points $S(-1,4)$ and $T(5,3)$.

5. $E(6,-3)$ and $F(4,7)$ are points on a straight line. Find the equation of the line.

6. $V(-3,7)$ and $D(1,8)$ are points on a straight line. Find the equation of the line in the form $ay + bx + c = 0$. State the value of a, b and c.

BEST BRAIN

B8.2.1.1.2 Draw the graph of a linear relation and use it to solve related problems.

In Basic 7, we learnt how to use a given scale to draw a graph for a given relation. In this section, we shall use the same concept to draw a graph and use the graph to determine subsequent missing elements.

Example 610

The mapping has a rule $y = 3x + 2$

x	-2	-1	0	1	2	3	4
y	—	-1	2	—	—	—	14

(a) (i) Copy and complete the spaces on the mapping.

(ii) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw 2 perpendicular axes OX and OY.

(iii) On the same graph sheet, mark the x-axis from -3 to 5 and y-axis from -6 to 16.

(b) Plot on the same graph sheet the ordered pair (x, y) from the mapping and join with a straight edge.

(c) Use the graph to find the value of

(i) y when $x = 2.5$,

(ii) x when $y = 14.6$.

Solution

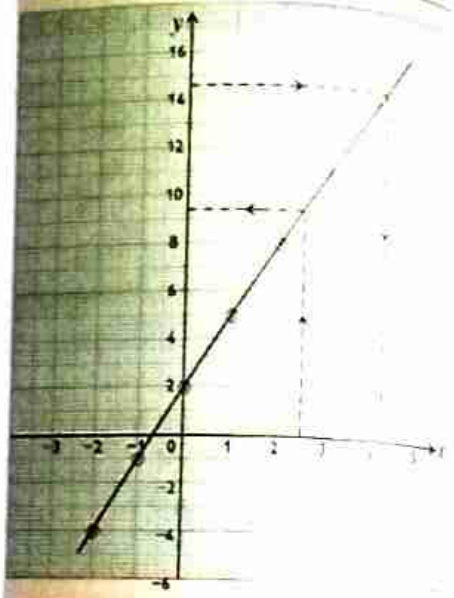
When $x = -2$
$y = 3(-2) + 2$
$= -6 + 2$
$= -4$

When $x = 1$
$y = 3(1) + 2$
$= 3 + 2$
$= 5$

When $x = 2$
$y = 3(2) + 2$
$= 6 + 2$
$= 8$

When $x = 3$
$y = 3(3) + 2$
$= 9 + 2$
$= 11$

x	-2	-1	0	1	2	3	4
y	-4	-1	2	5	8	11	14



(c) (i) When $x = 2.5$, $y = 9.4$

(ii) When $y = 14.6$, $x = 4.2$

Example 611

The table has a rule $y = a - 5x$

x	-3	-2	-1	0	1	2	3
y	30					10	

(a) (i) Find the value of a

(ii) Copy and complete the table

(iii) Using a scale of 2cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw 2 perpendicular axes OX and OY.

(iv) On the same graph sheet, mark the x-axis from -4 to 4 and y-axis from -10 to 35.

BEST BRAIN

(b) Plot on the same graph sheet the ordered pair (x, y) from the mapping and join with a straight edge.

(c) Use the graph to find the value of

(i) y when $x = 1.5$;

(ii) x when $y = 27$.

Solution

(a) (i) Given $x = 1$, $y = 10$
$y = a - 5x$
$10 = a - 5(1)$
$10 = a - 5$
$10 + 5 = a - 5 + 5$
$15 = a$

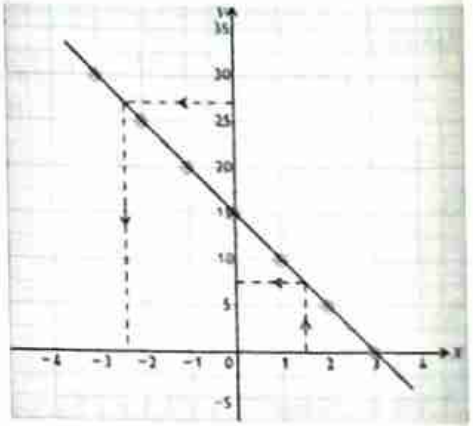
Or Given $x = -3$, $y = 30$
$y = a - 5x$
$30 = a - 5(-3)$
$30 = a + 15$
$30 - 15 = a + 15 - 15$
$15 = a$

(ii) The rule is $y = 15 - 5x$

When $x = -2$	When $x = -1$
$y = 15 - 5(-2)$	$y = 15 - 5(-1)$
$= 15 + 10$	$= 15 + 5$
$= 25$	$= 20$
When x = 0	When x = 2
$y = 15 - 5(-0)$	$y = 15 - 5(2)$
$= 15 - 0$	$= 15 - 10$
$= 15$	$= 5$

When x = 3
$y = 15 - 5(3)$
$= 15 - 15$
$= 0$

x	-3	-2	-1	0	1	2	3
y	30	25	20	15	10	5	0

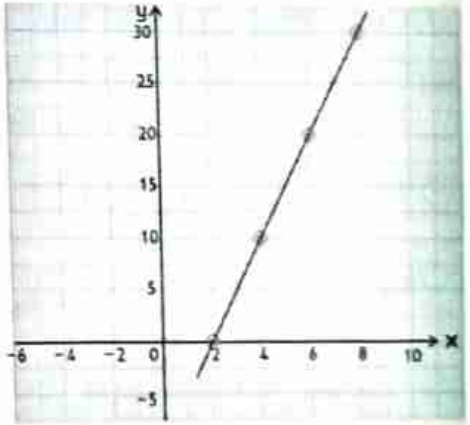


(c) (i) When $x = 1.5$, $y = 7.5$

(ii) When $y = 27$, $x = -2.4$

Example 612

The graph shows a linear relationship between x and y.



BEST BRAIN

(a) Write the scale used in the graph.
(b) Find the gradient of the line.
(c) Find the equation of the line.

Solution

(a) x-axis: 2 cm to 2 units, y-axis: 2 cm to 5 units.

(b) To find the gradient, choose any two points that lie on the line.
i.e. (4, 10) and (8, 30)

$$\text{Gradient} = \frac{30 - 10}{8 - 4} = \frac{20}{4}$$
$$= 5$$

(c) Let (x, y) represent any point on the line

$$5 = \frac{y - 10}{x - 4}$$
$$5(x - 4) = y - 10$$
$$5 \times x - 5 \times 4 = y - 10$$
$$5x - 20 = y - 10$$
$$5x - 20 + 10 = y - 10 + 10$$
$$5x - 10 = y$$

Therefore the equation of the line is $y = 5x - 10$

EXERCISE 176 Date

1. (a) Copy and complete the table for the relation $y = 2x + 2$

x	−2	−1	0	1	2	3	4	5
y			2					12

(b) Using a scale of 2cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw 2 perpendicular axes OX and OY.

(c) On the same graph sheet, mark the x-axis from −3 to 6 and y-axis from −10 to 14.

(d) Plot on the same graph sheet the ordered pair (x, y) from the table and join with a straight edge.

(e) Use the graph to find the value of
(i) x when $y = 7$;
(ii) y when $x = 3.4$.

2. The mapping has a rule $y = mx$ where m is a constant.

x	−1	0	1	2	3	4	5
y	—	0	—	—	—	12	—

(a) (i) Find the value of m.
(ii) Copy and complete the mapping
(iii) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw 2 perpendicular axes OX and OY.

(b) Plot on the same graph sheet the ordered pair (x, y) from the mapping and join with a straight edge.

(c) Use the graph to find the value of y when $x = 4.4$.

3. The table has a rule $y = 7 + ax$

x	−3	−2	−1	0	1	2	3	4
y	1			7				

(a) (i) Find the value of a.
(ii) Copy and complete the table.
(iii) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw 2 perpendicular axes OX and OY.

(b) Plot on the same graph the ordered pair (x, y) from the mapping and join with a straight edge.

BEST BRAIN

(c) Use the graph to find the value of
(i) y when $x = 1.5$;
(ii) x when $y = 8$.

4. The table has a rule $y = 5 - 2x$

x	−4	−3	−2	−1	0	1	2	3	4
y				7					−3

(a) (ii) Copy and complete the table.
(iii) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw 2 perpendicular axes OX and OY.

(b) Plot on the same graph the ordered pair (x, y) from the mapping and join with a straight edge.

(c) Use the graph to find the value of
(i) y when $x = 0.5$
(ii) x when $y = 8.8$

BEST BRAIN

B8.2.1.1.3A Use graphs of linear relation to solve real life problems

Most activities performed by humans can be modelled as a linear relation. In this section, we shall use a linear model from a real life situation to draw a line and interpret it.

Example 613

The distance covered by an athlete on his regular morning exercise is given by the relation $y = \frac{1}{2}t$ where y is the distance (in km) covered and t is the time (in hours)

t	0	2	4	6	8	10
y			2			

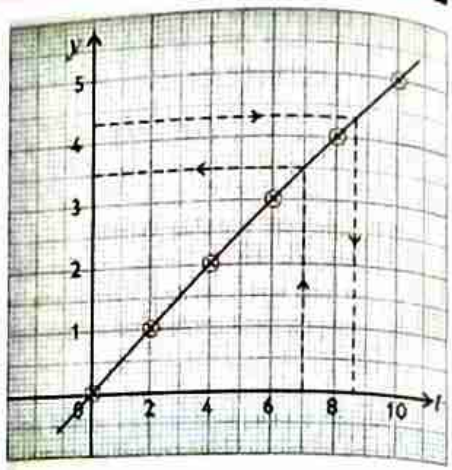
(a) Copy and complete the table.
(b) Using a scale of 2 cm to 1 unit on the y-axis and 2 cm to 2 units on the t-axis, draw 2 perpendicular axes OT and OY.
(c) Plot on the same graph sheet the ordered pair (t, y) and join with a straight edge.
(d) Use the graph to find the value of
(i) y when $t = 7$ hours;
(ii) t when $y = 4.3$ km.

Solution

(a) The relation is $y = \frac{1}{2}t$

When $t = 0$	When $t = 2$	When $t = 6$
$y = \frac{1}{2} \times 0$	$y = \frac{1}{2} \times 2$	$y = \frac{1}{2} \times 6$
$= 0$	$= 1$	$= 3$
When $t = 8$	**When $t = 10$**	
$y = \frac{1}{2} \times 8$	$y = \frac{1}{2} \times 10$	
$= 4$	$= 5$	

t	0	2	4	6	8	10
y	0	1	2	3	4	5



(d) (i) When $t = 7$ hours, $y = 3.5$ km
(ii) When $y = 4.3$ km, $t = 8.6$ hours

Example 614

The table below shows the weight of a new born baby monitored over a period of time in months.

t	0	1	2	3	4	5
w	1.5		4.5			

The child's weight is modelled by a linear rule $w = \frac{3}{2}t + 1.5$
(a) Copy and complete the table
(b) Using a scale of 2 cm to 1 unit on both axes, draw 2 perpendicular axes OT and OW
(c) Plot on the same graph sheet the ordered pair (t, w) and join with a straight edge.
(d) Use the graph to find the
(i) weight of the child after 4 and half months;
(ii) time after which the child will weigh 2.7 kg.

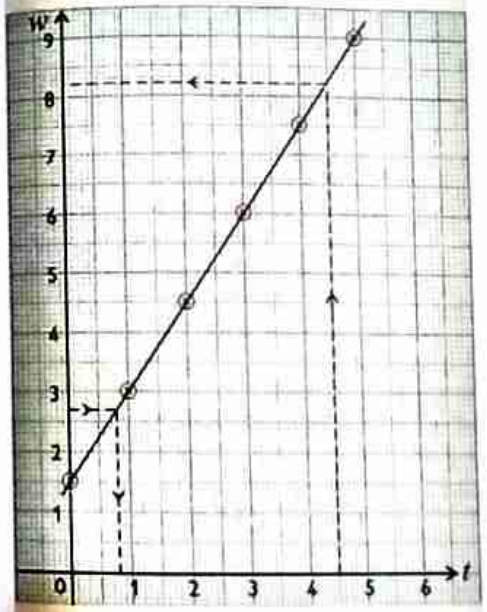
BEST BRAIN

Solution

(a) The relation is $w = \frac{3}{2}t + 1.5$

When $t = 1$	When $t = 3$
$y = \frac{3}{2} \times 1 + 1.5$	$y = \frac{3}{2} \times 3 + 1.5$
$= 1.5 + 1.5$	$= 4.5 + 1.5$
$= 3$	$= 6$
When $t = 4$	**When $t = 5$**
$y = \frac{3}{2} \times 4 + 1.5$	$y = \frac{3}{2} \times 5 + 1.5$
$= 6 + 1.5$	$= 7.5 + 1.5$
$= 7.5$	$= 9$

t	0	1	2	3	4	5
w	1.5	3	4.5	6	7.5	9



(d) (i) The weight of the child after 4 and half months 8.2 kg.
(ii) The time after which the child will weigh 2.7 kg is 0.8 months.

EXERCISE 177 Date

1. The amount of maize milled by a machine with time is given by the rule $y = 30t$ where y is in kilograms and t is in minutes.

t	0	5	10	15	20	25	30
y	0				600		

(a) Copy and complete the table
(b) Using a scale of 2 cm to 5 units on the t-axis and 2 cm to 100 units on the y-axis, draw 2 perpendicular axes OT and OY
(c) Plot on the same graph sheet the ordered pair (t, y) and join with a straight edge.
(d) Use the graph to find the
(i) quantity of maize milled after 28 minutes;
(ii) time spent to mill 800 kg.

2. Given that GH¢6.00 = \$1.00, copy and complete the table below relating cedis (x) to dollars (y).

x	12	24	30	36	42	48	54
y	2					8	

(a) Copy and complete the table
(b) Using a scale of 2 cm to 12 units on the x-axis and 2 cm to 1 unit on the y-axis, draw 2 perpendicular axes OX and OY
(c) Plot on the same graph sheet the ordered pair (x, y) and join with a straight edge.
(d) Use the graph to find the value of
(i) \$5.50 in cedis,
(ii) GH¢51.00 in dollars.

3. The table below shows the weight (y) of a new born baby monitored over a period of time (t) in months.

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t	0	1	2	3	4	5
y	2.1				8.1	

The child's weight is modelled by a linear rule $y = \frac{3}{2}t + 2.1$

(a) Copy and complete the table.

(b) Using a scale of 2 cm to 1 unit on both axes, draw 2 perpendicular axes OT and OY.

(c) Plot on the same graph sheet the ordered pair (t, y) and join with a straight edge.

(d) Use the graph to find the

(i) weight of the child after 2.8 months;

(ii) time after which the child will weigh 9.2 kg.

4. The amount paid on electricity usage is modelled by $c = \frac{3}{5}n + 10$ where c is the cost in cedis and n is the number of units consumed.

n	0	10	20	30	40	50	60
c	10		22				

(a) Copy and complete the table.

(b) Using a scale of 2 cm to 10 units on the n-axis and 2 cm to 5 units on the c-axis, draw 2 perpendicular axes ON and OC.

(c) Plot on the same graph sheet the ordered pair (n, c) and join with a straight edge.

(d) Use the graph to find the

(i) amount paid by Mr. Andoh for using 48 units of power;

(ii) units consumed by Appiadu if he paid GH¢30.00.

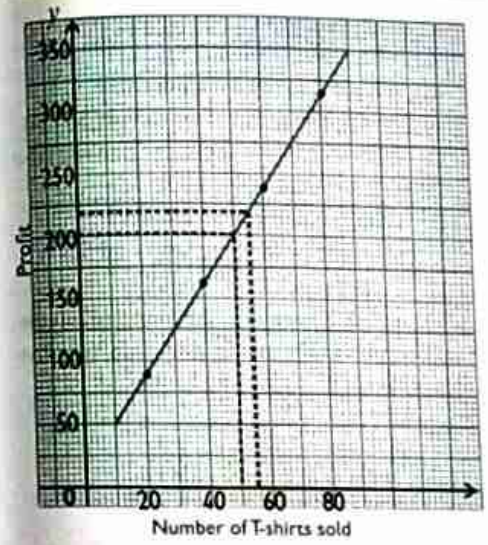
BEST BRAIN

B8.2.1.1.3B Use graphs of linear relation to solving real-life problems.

In real-life situations, you are sometimes presented with a graph to interpret. Therefore, let us use our knowledge of graphing linear relationships to interpret some graphs.

Example 615:

An online clothing company can earn GH¢4 for every t-shirt they sell.



a) How much can the company earn by selling 55 t-shirts?

b) How many t-shirts does the company need to sell so that they would earn GH¢200?

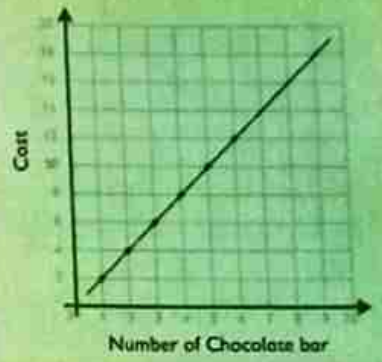
Solution

a) From the graph, we can see that 55 T-shirts corresponds to the selling price of GH¢217.50.

b) From the graph, we can see that selling price of GH¢200 corresponds to 50 T-shirts.

Example 616:

The graph below shows the cost of chocolate bars.



1. Create a table of values from the graph.
2. From the graph, estimate the cost of 9 chocolate bars.

Solution

a)

No of Chocolate bar	1	2	3	4	5	6
Cost GH¢	2	4	6	8	10	12

b) From the graph, 9 chocolate corresponds to GH¢18

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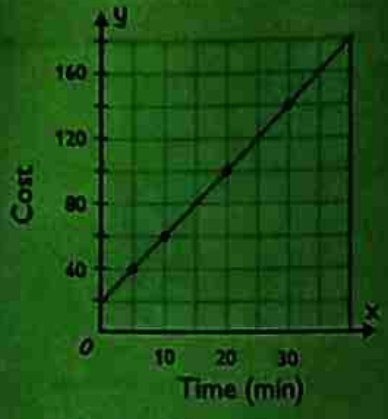
Example 617 The graph below shows the cost for time spent on a massaging machine.



a) From the graph, find the cost of spending 25 minutes on the machine.

b) How many minutes will a costumer spend on the machine for an amount of GH¢100?

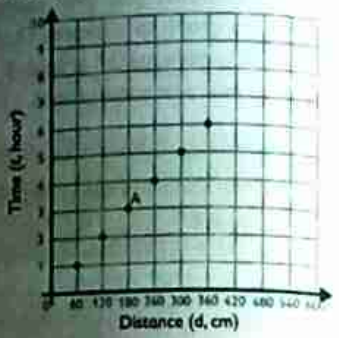
Solution

a) From the graph, 25 minutes on the machine correspond to GH¢120.

b) From the graph, GH100 correspond to 20 minutes.

EXERCISE 178 Date:

1. The graph shows the distance a car travels



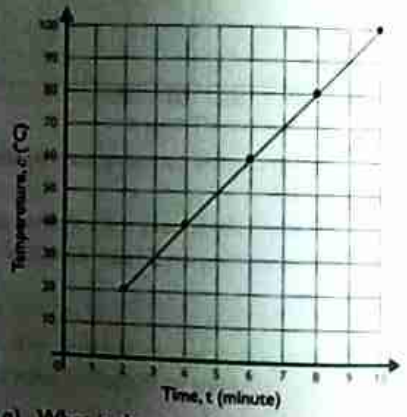
a) What does (180,3) represent?

b) What is the speed of the car at point A?

c) What distance will the car travel at the seventh hour?

d) Is the graph showing a linear relationship? Why or why not?

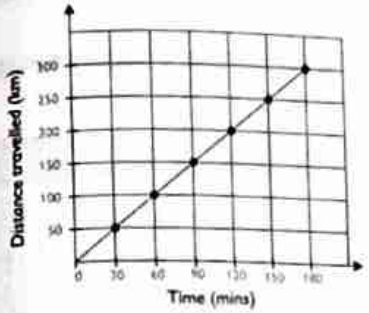
2. The graph shows the relationship of the temperature of water, c, and time, t, as the water boils.



a) What is the equation for the relationship?

b) Find the temperature of the water after 7 minutes of boiling.

3. The graph shows the distance a motorcyclist travels.

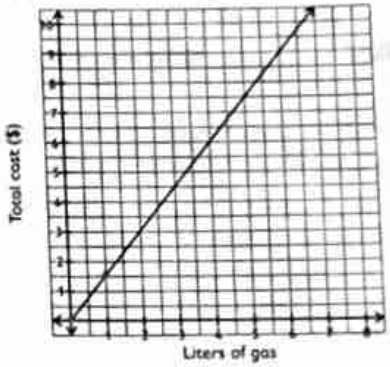


a) What is the distance covered at the 120th minute?

b) How much time will the cyclist use to cover a distance of 150km?

c) Is the graph showing a linear relationship? Why or why not?

4. The local gas station charges a certain rate for gas, as shown in the graph below



a) What is the price of gas at the gas station per litre?

b) How much will a customer pay for receiving 6 litres?

c) How many litres will a customer get for paying GH¢5?

STRAND 2.2

ALGEBRAIC EXPRESSIONS

B8.2.2.1 Use distributive property to solve multiplication of binomial expressions

In Basic 7, we learnt how to multiply algebraic expressions. In this section, we shall learn how to use distributive property to remove brackets and solve problems on multiplication of binomial expressions.

A binomial is an algebraic expression in a bracket that contains two terms. Expressions such as $(1 + 2x)$, $(a + b)$, $(3x - p)$, etc. are all examples of binomial expression.

Example 618

Expand and simplify the following.

(a) $3(5 + y)$ (b) $2a(b - 4)$

(c) $7(y - 5) - 3(y - 8)$

(d) $8 - (5m - 3) - 2(4 - 7m)$

Solution

(a) $3(5 + y) = 3 \times 5 + 3y$
$= 15 + 3y$

(b) $2a(b - 4) = 2a \times b + 2a \times (-4)$
$= 2ab - 8a$

(c) $7(y-5)-3(y-8) = 7xy+7\times(-5)-3xy-3\times(-8)$
$= 7y - 35 - 3y + 24$
$= 7y - 3y - 35 + 24$
$= 4y - 11$

(d) $8-(5m-3)-2(4-7m) = 8-1(5m-3)-2(4-7m)$
$= 8 - 1 \times 5m - 1 \times (-3) - 2 \times 4 - 2 \times (-7m)$
$= 8 - 5m + 3 - 8 + 14m$
$= 8 + 3 - 8 - 5m + 14m$
$= 3 + 9m$

Example 619

Expand and simplify the expression
$3m(5m - 2n) - 2n(3m + 7n)$

Solution

$3m(5m - 2n) - 2n(3m + 7n)$
$= 3m \times 5m + 3m \times (-2n) - 2n \times 3m - 2n \times 7n$
$= 3\times5\times m\times m+3\times-2\times m\times n-2\times3\times n\times m-2\times7\times n\times n$
$= 15m^2 - 6mn - 6mn - 14n^2$
$= 15m^2 - 12mn - 14n^2$

Example 620

Expand and simplify the binomial expression
$(a + 3)(a - 7)$

Solution

$(a + 3)(a - 7)$
$= a \times (a - 7) + 3(a - 7)$
$= a \times a + a \times (-7) + 3 \times a + 3 \times (-7)$
$= a^2 - 7a + 3a - 21$
$= a^2 - 4a - 21$

Example 621

Find the product of $(3a + 5b)$ and $(a + 2b)$

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Solution

$(3a + 5b) \times (a + 2b)$
$= 3a \times (a + 2b) + 5b \times (a + 2b)$
$= 3a \times a + 3a \times 2b + 5b \times a + 5b \times 2b$
$= 3a^2 + 6ab + 5ab + 10b^2$
$= 3a^2 + 11ab + 10b^2$

Example 622

Find $(3w + 2)^2$

Solution

$(3w + 2)^2 = (3w + 2) \times (3w + 2)$
$= 3w \times (3w + 2) + 2 \times (3w + 2)$
$= 3w \times 3w + 3w \times 2 + 2 \times 3w + 2 \times 2$
$= 9w^2 + 6w + 6w + 4$
$= 9w^2 + 12w + 4$

Example 623

Expand and simplify $(5a - 3b)^2$

Solution

$(5a - 3b)^2 = (5a - 3b) \times (5a - 3b)$
$= 5a \times (5a - 3b) - 3b \times (5a - 3b)$
$= 25a^2 - 15ab - 15ab + 9b^2$
$= 25a^2 - 30ab + 9b^2$

EXERCISE 179 Date

1. Expand and simplify each of the following.
   (a) $2(a + 3)$
   (b) $3x(y - 2)$
   (c) $-5p(2 - 3a)$
   (d) $(a + 3) + 2(a - 1)$
   (e) $5(2x + 3) - 4(3x - 5)$
   (f) $2x(3y - 7) + x(5y + 4)$
   (g) $2(a - 2b) - 3(2b - 4a)$
   (h) $(a + b + c) - 2(a - b + c)$
2. Find the product of $(x + 3)$ and $(2x - 1)$
3. Expand and simplify $(3a + 4)(2a - 3)$
4. Expand and simplify $(a + 2b)(a + 3b)$
5. Expand and simplify $(2a + 5)^2$
6. Expand and simplify $(x + y)^2$
7. Expand and simplify $(2a - 3b)^2$
8. Expand and simplify $(4a - \frac{1}{2})^2$

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B8.2.2.1.2 Perform addition, subtraction, multiplication and division of algebraic expressions including fractions

In Basic 7, we studied how to add, subtract, multiply and divide simple algebraic expressions. In this section, we shall first revise our knowledge on multiplication and division of algebraic expressions and solve some problems in algebraic expressions involving fractions.

Multiplication and division

In solving problems involving multiplication and division of simple algebraic expressions, we shall use our knowledge on powers of numbers.

Example 624

Simplify the following

(a) $3xy \times 5y^3$

(b) $a^3 \times 4ab \times 3a^2b^3$

(c) $\frac{1}{12}ab^3 \times 18a \times 4a^2b^3$

Solution

(a) $3xy \times 5y^3 = 3 \times 5 \times xy \times y^3$
$= 15xy^{1+3}$
$= 15xy^4$

(b) $a^3 \times 4ab \times 3a^2b^3 = 4 \times 3 \times a^3 \times ab \times a^2b^3$
$= 12 \times a^3 \times a \times a^2 \times b \times b^3$
$= 12a^{3+1+2}b^{1+3}$
$= 12a^6b^4$

(c) $\frac{1}{12}ab^3 \times 18a \times 4a^2b^3 = \frac{1}{12} \times 18 \times 4 \times ab^3 \times a \times a^2b^3$
$= \frac{18 \times 4}{12} \times a \times a \times a^2 \times b^3 \times b^3$
$= 6a^{1+1+2}b^{3+3}$
$= 6a^4b^6$

Example 625

Simplify the following

(a) $15a^3 \div 3a$

(b) $13a^2 \times 12a^3b^2 \div 6a^4b$

(c) $5x^3y \times 8x^2y^4 \div 60x^4y^3$

Solution

(a) $15a^3 \div 3a = \frac{15a^3}{3a^1}$
$= 5a^{3-1}$
$= 5a^2$

(b) $13a^2 \times 12a^3b^2 \div 6a^4b = \frac{13a^2 \times 12a^3b^2}{6a^4b}$
$= \frac{13 \times 12 \times a^{2+3}b^2}{6a^4b}$
$= \frac{13 \times 2 \times a^5b^2}{a^4b^1}$
$= 26a^{5-4}b^{2-1}$
$= 26ab$

(c) $5x^3y \times 8x^2y^4 \div 60x^4y^3 = \frac{5x^3y \times 8x^2y^4}{60x^4y^3}$
$= \frac{5 \times 8 \times x^{3+2} \times y^{1+4}}{60x^4y^3}$
$= \frac{8x^5y^5}{12x^4y^3}$
$= \frac{2}{3}x^{5-4}y^{5-3}$
$= \frac{2}{3}xy^2$

BEST BRAIN

EXERCISE 180 Date

Simplify each of the following

(a) $6p^3 \times 3pq^3$

(b) $xy \times 2xy \times 3x^2y$

(c) $\frac{1}{7}ab \times 3a^2b \times 21ab$

(d) $24m^4n^2 \div 6m^3n^2$

(e) $5a^2b^3 \times 4ab^2 \div 10a^3b^4$

(f) $42x^3y^4z^3 \div (7x^3y \times 3xy^2z^2)$

Addition and subtraction of algebraic fractions

To add or subtract algebraic fractions, we consider the steps below.

- Find the LCM of the denominators.
- Use the LCM to simplify the numerators.

Example 626

Simplify the following

(a) $\frac{a}{2} + \frac{a}{3}$ (b) $\frac{c+2d}{3} - \frac{3c}{5}$

(c) $\frac{2n}{5} - \frac{m-n}{2}$ (d) $\frac{a}{2} + \frac{a-2b}{3} - \frac{a+b}{4}$

Solution

(a) $\frac{a}{2} + \frac{a}{3} = \frac{3 \times a + 2 \times a}{6}$ (the LCM of 2 and 3 is 6)
$= \frac{3a + 2a}{6}$
$= \frac{5a}{6}$

(b) $\frac{c+2d}{3} - \frac{3c}{5} = \frac{5 \times (c+2d) - 3 \times 3c}{15}$ (the LCM of 3 and 5 is 15)
$= \frac{5c + 10d - 9c}{15}$
$= \frac{10d - 4c}{15}$

(c) $\frac{2n}{5} - \frac{m-n}{2} = \frac{2 \times 2n - 5 \times (m-n)}{10}$ (the LCM of 5 and 2 is 10)
$= \frac{4n - 5m + 5n}{10}$
$= \frac{4n + 5n - 5m}{10}$
$= \frac{9n - 5m}{10}$

(d) $\frac{a}{2} + \frac{a-2b}{3} - \frac{a+b}{4} = \frac{6 \times a + 4 \times (a-2b) - 3(a+b)}{12}$ (the LCM of 2, 3 and 4 is 12)
$= \frac{6a + 4a - 8b - 3a - 3b}{12}$
$= \frac{6a + 4a - 3a - 8b - 3b}{12}$
$= \frac{7a - 11b}{12}$

EXERCISE 181 Date

Simplify each of the following.

(a) $\frac{2a}{5} + \frac{a}{2}$ (b) $\frac{3b}{2} + \frac{b+1}{3}$

(c) $\frac{a+b}{7} - \frac{b}{2}$ (d) $\frac{2x+y}{4} - \frac{x-y}{2}$

(e) $\frac{7x}{3} - \frac{x+y}{2} - \frac{y-3x}{4}$ (f) $\frac{3x+4y}{5} - \frac{2x-y}{2}$

BEST BRAIN

Multiplication and division of algebraic fractions

When multiplying algebraic fractions, we multiply the numerators and the denominators separately.

To divide algebraic fraction of the form $\frac{a}{b} \div \frac{c}{d}$, we first change the division symbol to multiplication by taking the inverse of $\frac{c}{d}$ and multiply as used in multiplication of algebraic fractions.

Example 627

Simplify the following

(a) $\frac{ab}{8} \times \frac{32ab}{2b^2}$ (b) $\frac{12g^2h}{7g} \times \frac{35gh^3}{10gh}$

(c) $\frac{18}{cd} \div \frac{6}{c^2d^2}$ (d) $\frac{20ab^2c^3}{d} \div \frac{4b^2c^2}{3d^2}$

Solution

(a) $\frac{ab}{8} \times \frac{32ab}{2b^2} = \frac{32 \times ab \times ab}{8 \times 2 \times b^2}$

$= \frac{2a^2b^2}{b^2}$

$= 2a^2b^{2-2}$

$= 2a^2b^0$

$= 2a^2$

(b) $\frac{12g^2h}{7g} \times \frac{35gh^3}{10gh} = \frac{12 \times 35 \times g^2h \times gh^3}{7 \times 10 \times g \times gh}$

$= \frac{6g^3h^4}{g^2h^2}$

$= 6g^{3-2}h^{4-2}$

$= 6gh^2$

(c) $\frac{18}{cd} \div \frac{6}{c^2d^2} = \frac{18}{cd} \times \frac{c^2d^2}{6}$

$= \frac{18 \times c^2d^2}{6 \times cd}$

$= 3c^{2-1}d^{2-1}$

$= 3cd^2$

(d) $\frac{20ab^2c^3}{d} \div \frac{4b^2c^2}{3d^2}$

$= \frac{20ab^2c^3}{d} \times \frac{3d^2}{4b^2c^2}$

$= \frac{20 \times 3 \times a \times b^2c^3 \times d^2}{4 \times d \times b^2c^2}$

$= 5 \times 3 \times a \times b^{2-2} \times c^{3-2} \times d^{2-1}$

$= 15acd$

EXERCISE 182 Date

Simplify each of the following.

(a) $\frac{20}{ab} \div \frac{30}{ab^2}$

(b) $\frac{15x^2y^3}{10ym^2} \div \frac{2x^2y^2}{m^2}$

(c) $\frac{56a^2b^3}{14} \times \frac{a}{3a^2b^2}$

(d) $\frac{6m^2}{5n^2} \times \frac{3mn^2}{12mn}$

(e) $\frac{2}{3x^2y} \div \frac{4}{18xy^2}$

(d) $\frac{18mn^2r}{4n^2r^2} \times \frac{mn}{r^2} \div \frac{6m^2n}{8nr^2}$

B8.2.2.1.3 Substitute values to evaluate algebraic expressions including fractions

In Basic 7, we learnt how to evaluate algebraic expressions when the values for each variable in the expression are given. In this section, we shall use the same concept to evaluate given expression including fractions by first simplifying the expression before substituting the values to evaluate.

Example 628

Write $5ab^2 \times ab \times 3a$ in its simplified form and substitute the values to evaluate, given that $a = \frac{1}{2}$ and $b = 2$.

Solution

$5ab^2 \times ab \times 3a = 5 \times 3 \times ab^2 \times ab \times a$

$= 15a^3b^3$

$= 15 \times \left(\frac{1}{2}\right)^3 \times 2^3$

$= 15 \times \frac{1^3}{2^3} \times 2^3$

$= 15 \times 1$

$= 15$

Example 629

Write $\frac{18e^2f^3}{10mn} \div \frac{18e^3f^4}{25mn^3}$ in its simplified form and substitute the values to evaluate, given that $n = -1$ and $f = 3$.

Solution

$\frac{18e^2f^3}{10mn} \div \frac{6e^3f^4}{25mn^3} = \frac{18e^2f^3}{10mn} \times \frac{25mn^3}{6e^3f^4}$

$= \frac{18^3 \times 25^5 \times e^2f^3 \times mn^3}{10^2 \times 6^2 \times mn \times e^3f^4}$

$= \frac{15n}{2f}$

$\frac{18e^2f^3}{10mn} \div \frac{6e^3f^4}{25mn^3} = \frac{15 \times (-1)}{2 \times (3)}$

$= \frac{-15}{6}$

$= -\frac{5}{2}$

$= -2\frac{1}{2}$

Example 630

Write $\frac{10am + 15an}{4n} \times \frac{6n^2}{2m + 3n}$ in its simplified form and substitute the values to evaluate it given that $a = 2$ and $n = \frac{1}{2}$.

Solution

$\frac{10am+15an}{4n} \times \frac{6n^2}{2m+3n} = \frac{5a(2m+3n)}{4n} \times \frac{16n^3}{(2m+3n)}$

$= \frac{5a(2m+3n)}{4n} \times \frac{16n^3}{(2m+3n)}$

$= \frac{5a \times 16n^3}{4n}$

$= 5a \times 4 \times n$

$= 20an$

$= 20(2)\left(\frac{1}{2}\right)$

$= \frac{40}{2}$

$= 20$

Example 631

Evaluate $\frac{5}{2x-1} - \frac{1}{x+3}$ given that $x = 2$

Solution

$\frac{5}{2x-1} - \frac{1}{x+3} = \frac{5}{2 \times 2 - 1} - \frac{1}{2+3}$

$= \frac{5}{4-1} - \frac{1}{5}$

$$\frac{5}{2x-1} - \frac{1}{x+3} = \frac{5}{3} - \frac{1}{5}$$
$$= \frac{5\times 5 - 1\times 3}{15}$$
$$= \frac{25-3}{15}$$
$$= \frac{22}{15}$$
$$= 1\frac{7}{15}$$

EXERCISE 183 Date

1. Evaluate $3x^2 + 4x - 5$ given that $x = 1$.
2. Evaluate $\frac{1}{x} + \frac{x}{2x+1}$ given that $x = 2$.
3. Evaluate $\frac{x+y}{2x+y} + \frac{x}{y}$ given that $x = 1$ and $y = 2$.
4. Write $\frac{10ab}{9} \times \frac{27}{5abc}$ in its simplified form and substitute the value to evaluate, given that $c = 2$.
5. Write $\frac{3ab^2}{8xy} \times \frac{16xy^2}{21ab^3}$ in its simplified form and substitute the values to evaluate, given that $b = 1$ and $y = 2$.
6. Write $\frac{4ax + 8x}{3x - 1} \div \frac{a+2}{6x-2}$ in its simplified form and substitute the value to evaluate, given that $x = -1$ and $a = 2$.
7. Write $\frac{36m^4n^3}{8m} \div \frac{n^2m^2}{4mn^3}$ in its simplest form and substitute the values to evaluate, given that $m = 3$ and $n = \frac{1}{3}$
8. Write $\frac{3x^2 + 6x}{2xy + 4y} \times \frac{2y}{9x^2}$ in its simplified form and substitute the value to evaluate, given that $x = 1$ and $y = 2$.

BEST BRAIN

B8.2.2.1.4 Factorise given expressions involving the four operations

Factorisation by common factors

Consider the expression $2 + 4y$. We can observe that this expression can be written as
$2\times 1 + 2\times 2y$.
2 is common to 2×1 and $2\times 2y$.
We can therefore factorise 2 out to get $2(1+2y)$.
To factorise a given expression, we always look for a common factor, bring it out of the expression (factorise) and express the other factors in a bracket.

Example 632
Factorise $by - cy$

Solution
$by - cy = b\times y - c\times y$
$= y(b - c)$

Example 633
Factorise $26m - 39n$

Solution
$26m - 39n = 13\times 2m - 13\times 3n$
$= 13(2m - 3n)$

Example 634
Factorise $6s - 8st$

Solution
$6s - 8st = 2\times 3s - 2\times 4st$
$= 2s\times 3 - 2s\times 4t$
$= 2s(3 - 4t)$

Example 635
Factorise $12ab^3 - 15a^2b$

Solution
$12ab^3 - 15a^2b = 3\times 4\times a\times b\times b - 3\times 5\times a\times a\times b$
$= 3ab(4b - 5a)$

Example 636
Factorise $5\times 8.36 + 5\times 3.64$

Solution
$5\times 8.36 + 5\times 3.64 = 5(8.36 + 3.64)$
$= 5(12)$
$= 60$

Example 637
Factorise $7.52\times 14 - 5.52\times 14$

Solution
$7.52\times 14 - 5.52\times 14 = 14(7.52 - 5.52)$
$= 14(2)$
$= 28$

EXERCISE 184 Date

1. Factorise each of the following
   (a) $ah + ak$
   (b) $15g - 9m$
   (c) $14kw + 6k$
   (d) $18st - 27st^2$
   (e) $3x^2 - 12x$
   (f) $5y - y^3$
   (g) $7ab - 21ab^2$
   (h) $2y^3 - 14xy^2$

BEST BRAIN

2. Evaluate each of the following:
   (a) $7 \times 9.32 + 7 \times 0.68$
   (b) $24 \times 3.89 - 1.89 \times 24$
   (c) $8 \times 0.4 + 8 \times 0.32 + 8 \times 0.28$
   (d) $18 \times 0.72 + 18 \times 0.68 + 18 \times 0.60$

Factorisation by grouping

Usually, factorisation by grouping involves four terms. To factorise by grouping, we put the expression into groups of two such that each group has a common factor. Let us use the examples below to explain how to factorise by grouping.

Example 638

Factorise $ab + 2a + b + 2$

Solution

We can observe that the first two expressions $ab + 2a$ has a common factor of a and the last two expressions also have a common factor of 1.

$$\begin{aligned} ab + 2a + b + 2 &= (ab + 2a) + (b + 2) \\ &= a(b + 2) + 1(b + 2) \\ &\quad (b + 2) \text{ is also common} \\ &= (b + 2)(a + 1) \end{aligned}$$

Example 639

Factorise $2ab + 2a + 3b + 3$

Solution

$$\begin{aligned} 2ab + 2a + 3b + 3 &= (2ab + 2a) + (3b + 3) \\ &= 2a(b + 1) + 3(b + 1) \\ &\quad (b + 1) \text{ is common} \\ &= (b + 1)(2a + 3) \end{aligned}$$

Example 640

Factorise $6xy - 4x + 9y - 6$

Solution

$$\begin{aligned} 6xy - 4x + 9y - 6 &= (6xy - 4x) + (9y - 6) \\ &= 2x(3y - 2) + 3(3y - 2) \\ &\quad (3y - 2) \text{ is common} \\ &= (3y - 2)(2x + 3) \end{aligned}$$

Example 641

Factorise $14m^2 + nz - 2mz - 7nm$

Solution

We can observe that the first two terms $14m^2 + nz$ don't have any common factor so we need to rearrange the expression.

$$\begin{aligned} &14m^2 + nz - 2mz - 7nm \\ &= 14m^2 - 7mn - 2mz + nz \\ &= (14m^2 - 7mn) + (2mz + nz) \\ &= 7m(2m - n) - z(2m - n) \\ &= (2m - n)(7m - z) \end{aligned}$$

Example 642

Factorise $6an + 2n - 3a - 1$

Solution

$$\begin{aligned} 6an + 2n - 3a - 1 &= (6an + 2n) + (-3a - 1) \\ &= 2n(3a + 1) - 1(3a + 1) \\ &= (3a + 1)(2n - 1) \end{aligned}$$

EXERCISE 185 Date

Factorise the following expressions.

(a) $2a+ad+2+d$ (b) $3xy+3y+4x+4$

(c) $2gh-6g+4h-12$ (d) $5dm+10m+7d+14$

(e) $60a^2b-10a+12ab^2-2b^2$ (f) $4k-2hk-6+3h$

(g) $15f^2+xy+3fy+5xf$ (h) $16mn-4m+8n-2$

STRAND 2.3

VARIABLES AND EQUATIONS

B8.2.3.1.1 Translate word problems into linear inequalities in one variable and vice versa

From Basic 4, we started comparing numbers using the symbol such as greater than (>) or less than (<). In this section, we shall use the symbols $>, <, \geq$ or $\leq$ to write a number sentence and vice versa.

Example 643

Write a statement using algebraic inequality.
"Appiadu scored y marks in a mathematics test and failed. If the pass mark for the test was 50."

Solution

Algebraic inequality: $y < 50$

Example 644

Write a statement using algebraic inequality.
"Linda went to a shop with GH₵80.00 to buy a dress which cost m cedis." Her money was enough to buy the dress.

Solution

Algebraic inequality: $m \geq 80$

Example 645

Write a statement using algebraic inequality.
"The sum of twice a certain number and 3 is greater than 18."

Solution

Let the number = g
Sum of twice the number and 3 is $2g + 3$
Algebraic inequality: $2g + 3 > 18$

Example 646

Write a statement for the algebraic inequality $x > 13$.

Solution

Note that one can write any statement from the given inequality.
"Addo is older than Akufo but Akufo is 13 years and Addo is x years."

Example 647

Write a statement for the algebraic inequality $3x < 24$.

Solution

"3 times a certain number is less than 24."

Example 648

Write a statement for the algebraic inequality $d - 5 > 29$.

Solution

"The difference between d and 5 is greater than 29."

EXERCISE 186 Date

1. Write the statement for each of the following using algebraic inequality.
   (a) Yeboah is taller than Nkansah and Nkansah is 157 cm. The height of Yeboah is y cm.
   (b) Ankamah passed a test with m marks and the pass mark for the test was 70.
   (c) Linda went to school with 12 exercise books but John had less number of exercise books than Linda. John had x exercise books.

2. Write a statement for each of the following algebraic inequalities.
   (a) $2x + 1 > 7$
   (b) $\frac{1}{2}h < 15$
   (c) $20 - 3x < 5$
   (d) $k - 3 > 11$

BEST BRAIN

B8.2.3.1.2 Solve simple linear inequality

In this section, we shall use our knowledge on how to use balancing linear equations to solve simple linear inequalities.

Example 649

Solve $x - 2 > 7$

Solution

$x - 2 > 7$
$x - 2 + 2 > 7 + 2$
$x > 9$

Example 650

Solve $13 \leq 7 + y$

Solution

$13 \leq 7 + y$
$13 - 7 \leq 7 - 7 + y$
$6 \leq y$

Note that $6 \leq y$ can also be written as $y \geq 6$.

Example 651

Solve $5x - 7 \leq 8$

Solution

$5x - 7 \leq 8$
$5x - 7 + 7 \leq 8 + 7$
$5x \leq 15$
$\frac{5x}{5} \leq \frac{15}{5}$
$x \leq 3$

Example 652

Solve $3x + 5 > x + 11$

Solution

$3x + 5 > x + 11$
$3x - x + 5 > x - x + 11$
$2x + 5 > 11$
$2x + 5 - 5 > 11 - 5$
$2x > 6$
$\frac{2x}{2} > \frac{6}{2}$
$x > 3$

Example 653

Solve $15 \geq 6 + 3x$

Solution

$15 \geq 6 + 3x$
$15 - 6 \geq 6 - 6 + 3x$
$9 \geq 3x$
$\frac{9}{3} \geq \frac{3x}{3}$
$3 \geq x$

Example 654

Solve $17 < 9 - 2x$

Solution

$17 < 9 - 2x$
$17 + 2x < 9 - 2x + 2x$
$17 + 2x < 9$
$17 - 17 + 2x < 9 - 17$
$2x < -8$
$\frac{2x}{2} < -\frac{8}{2}$
$x < -4$

EXERCISE 187 Date

Solve the following linear inequalities

(a) $y + 4 > 9$
(b) $15 < 3 + x$
(c) $3x - 4 \leq 14$
(d) $7 + 2m \geq 15$
(e) $3x + 6 > x + 18$
(f) $19 < 11 - 4k$
(g) $15 - x < 4x + 5$
(h) $12 - 3g \geq 3$

BEST BRAIN

B8.2.3.1.3 Determine solution sets of simple linear inequalities in given domains

Consider the inequality $x \leq 3$, if x is a set of positive whole numbers, then the domain of x is $\{1, 2, 3, 4, \ldots\}$

With this given domain, we can find the solution set that satisfies x as $\{1,2,3\}$.

In this section, we shall determine solution set of simple linear inequalities when the domain is given.

Example 655

Find the solution set of the inequality $x - 3 \leq 3x - 13$ if x has a domain $\{3, 4, 5, 6, 7, 8\}$

Solution

$$x - 3 \leq 3x - 13$$
$$x - x - 3 \leq 3x - x - 13$$
$$-3 \leq 2x - 13$$
$$-3 + 13 \leq 2x - 13 + 13$$
$$10 \leq 2x$$
$$2x \geq 10$$
$$\frac{2x}{2} \leq \frac{10^5}{2}$$
$$x \geq 5$$

Solution set $\{5, 6, 7, 8\}$

Example 656

Find the solution set of the inequality $5y + 7 < -2y + 21$ if y has a domain $\{1, 2, 3, 5, 6\}$

Solution

$$5y + 7 < -2y + 21$$
$$5y + 2y + 7 < -2y + 2y + 21$$
$$7y + 7 < 21$$
$$7y + 7 - 7 < 21 - 7$$
$$7y < 14$$
$$\frac{7y}{7} < \frac{14^2}{7}$$
$$y < 2$$

Solution set $\{1\}$

EXERCISE 188 Date

1. Find the solution set of the inequality $4 - 3n \geq 2n + 7$ if n has a domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$.
2. Find the solution set of the inequality $2x + 3 < 4x - 11$ if x has a domain $\{3, 4, 5, 6, 7, 8, 9, 10\}$.
3. Find the solution set of the inequality $3w - 2w \leq 7 + 3$ if w has a domain $\{6, 7, 8, 9, 10, 11, 12, 13\}$.
4. Find the solution set of the inequality $3 - 4k + 8 > 2k - 13$ if k has a domain {factors of 24}.
5. Find the solution set of the inequality $6m - 3 \geq 3m + 9$ if m has a domain {first 5 multiples of 3}.
6. Find the solution set of the inequality $\frac{1}{2}x + (x + 1) > 7$ if x has a domain {multiples of 2 less than 10}.
7. Find the solution set of the inequality $\frac{1}{2}(1 - 2x) + \frac{1}{3}(2x - 1) > -1$ if x has a domain $\{0, 1, 2, 3, 4, 5\}$.

STRAND 3: GEOMETRY AND MEASUREMENT

SUB-STRANDS

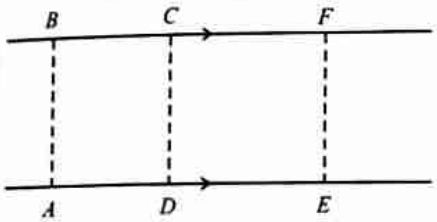
- Shapes and space
- Measurement
- Position and transformation

STRAND 3.1

SHAPES AND SPACE

B8.3.1.1.1 Demonstrate understanding of the properties of parallel lines

Look at the diagram.

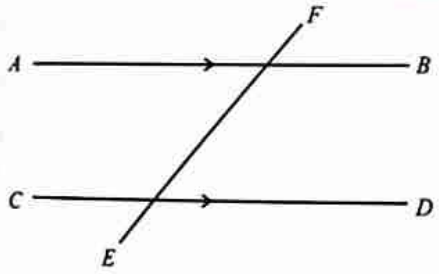


We can observe that distance $\overline{AB}$, $\overline{DC}$ and $\overline{EF}$ are the same.

Parallel lines are lines that maintain the same distance between them. This means that if the lines are to be extended, they will never meet.

We use the symbol '//' to show parallel lines.

Transversal

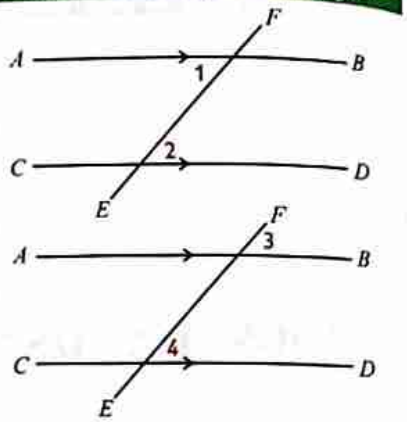


Line EF is called transversal

A line that intersects two or more lines in a plane at different points is called a transversal.

Line EF forms 8 angles on lines AB and CD.

Alternate and corresponding angles

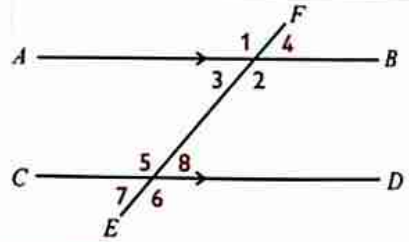


$m\angle 1 = m\angle 2$ (alternate angles)

$m\angle 3 = m\angle 4$ (corresponding angles)

$m\angle$ means measure of angle.

Vertical angles and linear pair



Vertical angles

$m\angle 1 = m\angle 2$, $m\angle 3 = m\angle 4$, $m\angle 5 = m\angle 6$ and $m\angle 7 = m\angle 8$. These pairs of angles are called vertically opposite angles.

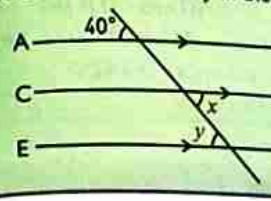
BEST BRAIN

Linear pair (angles on a straight line)

$m\angle 1 + m\angle 4 = 180°$, $m\angle 3 + m\angle 2 = 180°$, $m\angle 5 + m\angle 8 = 180°$ and $m\angle 6 + m\angle 7 = 180°$.

Example 657

Find the values of x and y in the figure.



Solution

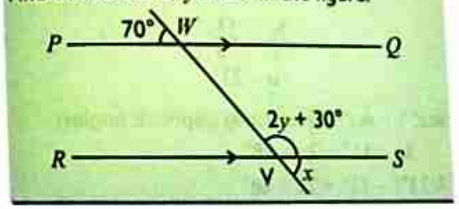
$m\angle y = 40°$ (corresponding angles)

$m\angle x = m\angle y$ (alternate angles)

$m\angle x = 40°$

Example 658

Find the values of y and x in the figure.



Solution

$\angle RVW = 70°$ (corresponding angles)

$m\angle x = \angle RVW$ (vertically opposite angles)

$m\angle x = 70°$

$x + 2y + 30° = 180°$ (angles on a straight line)

$70° + 2y + 30° = 180°$

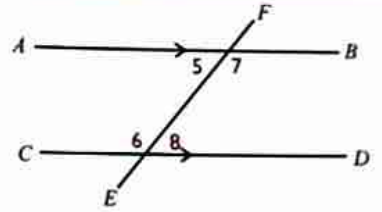
$2y = 180° - 100°$

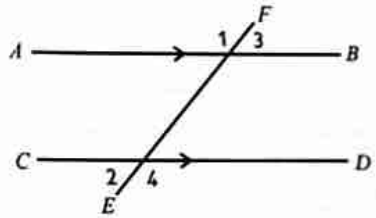
$2y = 80°$

$\frac{2y}{2} = \frac{80°}{2}$

$y = 40°$

Consecutive interior and exterior angles

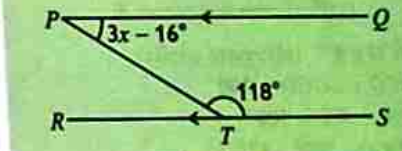




$m\angle 5 + m\angle 6 = 180°$ and $m\angle 7 + m\angle 8 = 180°$ (consecutive interior angles)

$m\angle 1 + m\angle 2 = 180°$ and $m\angle 3 + m\angle 4 = 180°$ (consecutive exterior angles)

Example 659



In the diagram, $\overline{QP}$ and $\overline{SR}$ are parallel.

$\angle QPT = 3x - 16°$ and $\angle PTS = 118°$.

Find the value of x.

BEST BRAIN

Solution

$\angle QPT + \angle PTS = 180°$ (consecutive interior angles)

$3x - 16° + 118° = 180°$

$3x + 102° = 180°$

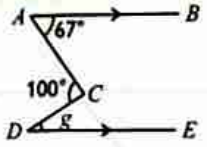
$3x = 180° - 102°$

$3x = 78°$

$\frac{3x}{3} = \frac{78°}{3}$

$x = 26°$

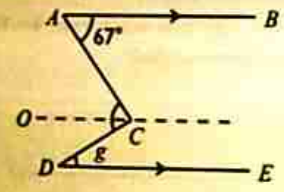
Example 660



In the diagram, $\overline{AB}$ and $\overline{DE}$ are parallel, $\angle BAC = 67°$, $\angle ACD = 100°$ and $\angle CDE = g$. Find the value of g.

Solution

Draw a line parallel to $\overline{AB}$ and $\overline{DE}$



$\angle ACO = 67°$ (alternate angles)

$\angle ACO + \angle OCD = 100°$

$67° + \angle OCD = 100°$

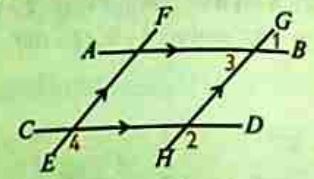
$\angle OCD = 100° - 67°$

$\angle OCD = 33°$

$g = \angle OCD$ (alternate angles)

$g = 33°$

Example 661



In the diagram, $\overline{EF} \parallel \overline{HG}$ and $\overline{CD} \parallel \overline{HG}$. Given that $m\angle 1 = 3a - 15°$, $m\angle 3 = 2x + 28°$, $m\angle 2 = 80° + 2a$ and $m\angle 4 = 7y + 42°$. Find the values of a, x and y.

Solution

From the diagram,

$m\angle 1 = m\angle 3$ (vertically opposite angles)

$m\angle 1 + m\angle 2 = 180°$ (consecutive exterior angles)

$m\angle 2 = m\angle 4$ (corresponding angles)

$m\angle 1 + m\angle 2 = 180°$

$3a - 15° + 80° + 2a = 180°$

$3a + 2a - 15° + 80° = 180°$

$5a + 65° = 180°$

$5a = 180° - 65°$

$5a = 115°$

$\frac{5a}{5} = \frac{115°}{5}$

$a = 23°$

$m\angle 1 = m\angle 3$ (vertically opposite angles)

$3a - 15° = 2x + 28°$

$3(23°) - 15° = 2x + 28°$

$69° - 15° = 2x + 28°$

$54° = 2x + 28°$

$54° - 28° = 2x$

$26° = 2x$

$\frac{26°}{2} = \frac{2x}{2}$

$13° = x$

BEST BRAIN

$m\angle 2 = m\angle 4$ (corresponding angles)

$80° + 2a = 7y + 42°$

$80° + 2(23°) = 7y + 42°$

$80° + 46° = 7y + 42°$

$126° = 7y + 42°$

$126° - 42° = 7y$

$84° = 7y$

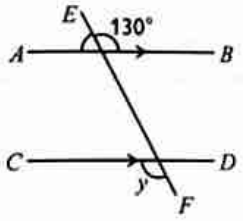
$\frac{84°}{7} = \frac{7y}{7}$

$12° = y$

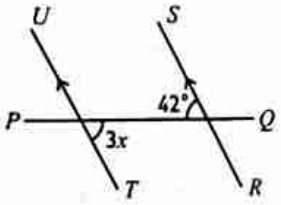
EXERCISE 189 Date

1. Find the values of the letters in each of the following diagrams.

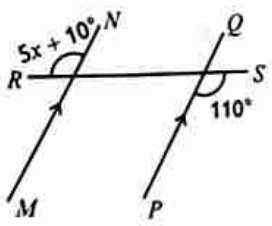
(a)



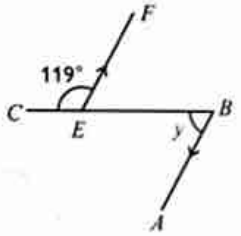
(b)



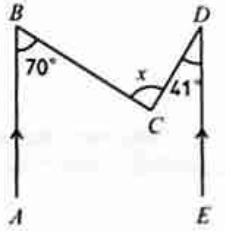
(c)



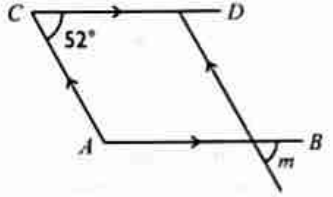
(d)



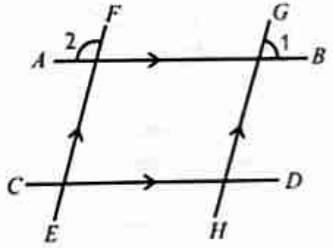
(e)



(f)

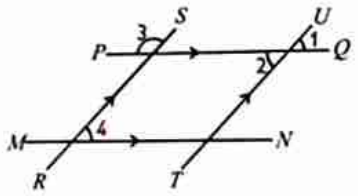


2. In the diagram, $\overline{AB} \parallel \overline{CD}$ and $\overline{EF} \parallel \overline{HG}$. Given that $m\angle 1 = 3x + 20°$ and $m\angle 2 = x + 96°$. Find the values of x.



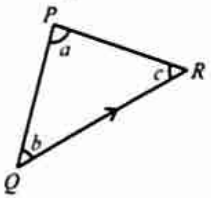
BEST BRAIN

3. In the diagram, $\overline{MS} \parallel \overline{PQ}$ and $\overline{RS} \parallel \overline{TU}$.
Given that $m\angle 1 = 2x - 8°$, $m\angle 2 = 5a - 6°$, $m\angle 3 = 88° + 3b$ and $m\angle 4 = 4x - 30°$.
Find the values of a, b and x.

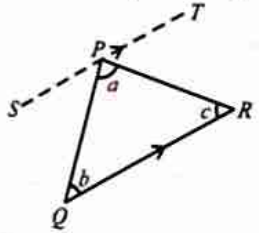


B8.3.1.1.2A Demonstrate understanding of angle sum property of a triangle

Consider the triangle.



To find the sum of the interior angles of triangle PQR, we shall draw a line parallel to $\overline{QR}$.

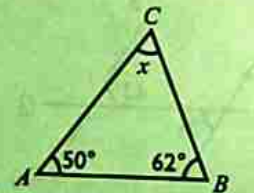


$\angle SPQ = b$ (alternate angles)
$\angle TPR = c$ (alternate angles)
$\angle QPR = a$
But $\angle SPQ + \angle QPR + \angle TPR = 180°$
(angles formed on a straight line)

$a + b + c = 180°$
Angle property of a triangle states that the sum of the interior angles of a triangle add up to 180°.

Example 662

Find the value of x in the diagram.

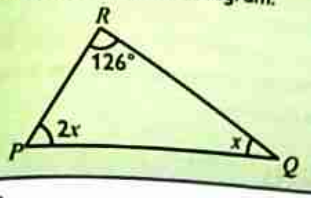


Solution

$50° + 62° + x = 180°$
(sum of the interior angles of a triangle)
$112° + x = 180°$
$x = 180° - 112°$
$x = 68°$

Example 663

Find the value of x in the diagram.

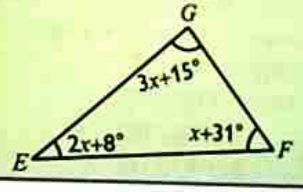


Solution

$2x + x + 126° = 180°$
(sum of the interior angles of a triangle)
$3x + 126° = 180°$
$3x = 180° - 126°$
$3x = 54°$
$\frac{3x}{3} = \frac{54°}{3}$
$x = 18°$

Example 664

Find the value of x in the diagram.



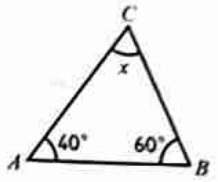
Solution

$2x + 8° + x + 31° + 3x + 15° = 180°$
(sum of the interior angles of a triangle)
$2x + x + 3x + 8° + 31° + 15° = 180°$
$6x + 54° = 180°$
$6x = 180° - 54°$
$6x = 126°$
$\frac{6x}{6} = \frac{126°}{6}$
$x = 21°$

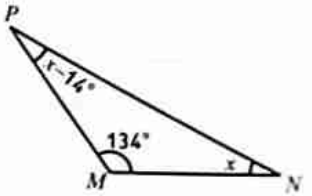
EXERCISE 190 Date

Find the value of x in the diagram.

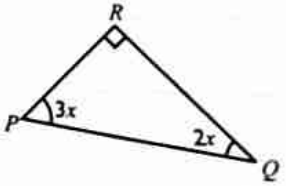
(a)



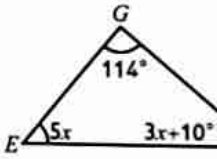
(b)



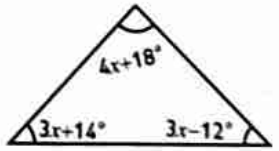
(c)



(d)

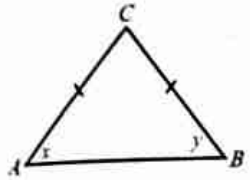


(e)



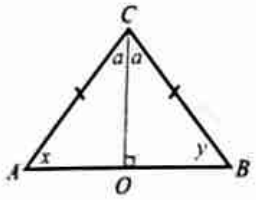
BS 3.1.1.28 Demonstrate understanding of angle property of an isosceles triangle

From Basic 5, we learnt that an isosceles triangle is a triangle that has two of its sides equal. In this section, we shall establish the angle property of an isosceles triangle. Consider triangle ABC below.



From the triangle, $\overline{AC} = \overline{BC}$ forming an angle at C

Bisect $\angle ACB$ to meet $\overline{AB}$ at 90° (by construction)



From ΔOBC

$a + 90° + y = 180°$

$a + y = 180° - 90°$

$a + y - y = 90 - y°$

$a = 90° - y$

From ΔOAC

$a + 90° + x = 180°$

$a + x = 180° - 90°$

$a + x = 90°$

But $a = 90° - y$

$90° - y + x = 90°$

$-y + x = 90° - 90°$

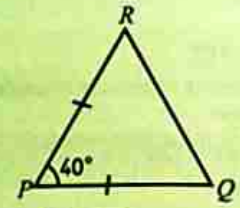
$-y + y + x = y$

$x = y$

This means that $\angle OAC = \angle OBC$

The angle property of an isosceles triangle states that the angles opposite the two equal sides are the same.

Example 665



In the triangle $\overline{PR} = \overline{PQ}$, $\angle QPR = 40°$, find $\angle PRQ$.

Solution

$40° + \angle PRQ + \angle RQP = 180°$
(sum of the interior angles of a triangle)
But $\angle PRQ = \angle RQP$
(angle property of isosceles triangle)

$40° + \angle PRQ + \angle PRQ = 180°$

$40° + 2\angle PRQ = 180°$

$2\angle PRQ = 180° - 40°$

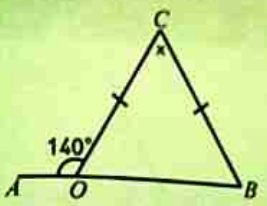
$2\angle PRQ = 140°$

$\frac{2\angle PRQ}{2} = \frac{140°}{2}$

$\angle PRQ = 70°$

BEST BRAIN

Example 666



In the triangle $\overline{OC} = \overline{BC}$, $\angle AOC = 140°$, find x.

Solution

$140° + \angle BOC = 180°$ (angles on a straight line)

$\angle BOC = 180° - 140°$

$= 40°$

$\angle OBC = \angle BOC$
(angle property of isosceles triangle)

$\angle OBC = 40°$

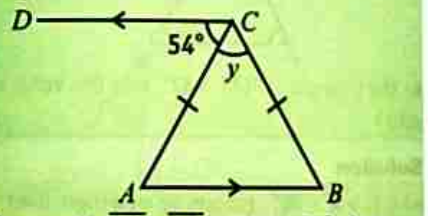
$40° + 40° + x = 180°$
(sum of the interior angles of a triangle)

$80° + x = 180°$

$x = 180° - 80°$

$x = 100°$

Example 667



In the triangle $\overline{AC} = \overline{BC}$, $\angle DCA = 54°$, find y.

Solution

$\angle CAB = \angle DCA$ (alternate angles)

$\angle CAB = 54°$

$\angle ABC = \angle CAB$ (angle property of isosceles triangle)

$= 54°$

$54° + 54° + y = 180°$
(sum of the interior angles of a triangle)

$108° + y = 180°$

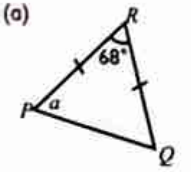
$y = 180° - 108°$

$y = 72°$

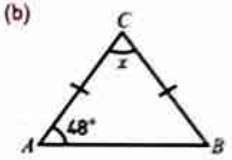
EXERCISE 191 Date

Find the values of the letters in each of the following diagrams.

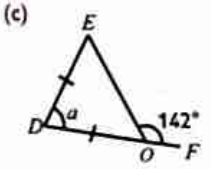
(a)



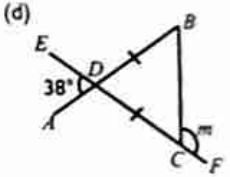
(b)



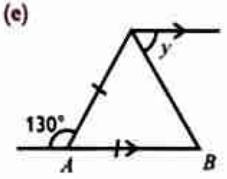
(c)



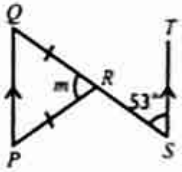
(d)



(e)

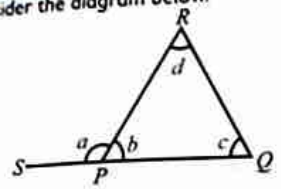


(f)



B8 3.1.1.2C Demonstrate understanding of angle property of an exterior angle of a triangle

Consider the diagram below.



$\angle SPR = a$ is called an exterior angle of triangle PQR.

$a + b = 180°$ (angles on a straight line)

$a - a + b = 180° - a$

$b = 180° - a$

$b + c + d = 180°$

(sum of the interior angles of a triangle)

$180° - a + c + d = 180°$

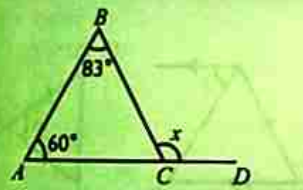
$-a + c + d = 180° - 180°$

$-a + a + c + d = +a$

$c + d = a$

The exterior angle property of a triangle states that the exterior angle of a triangle is the same as the sum of the two interior opposite angles.

Example 668



In the triangle $\angle BAC = 60°$ and $\angle ABC = 83°$, find x.

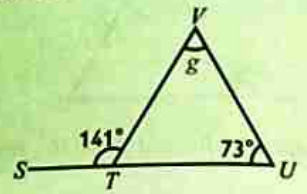
Solution

$60° + 83° = x$

(exterior angle property of a triangle)

$143 = x°$

Example 669



In the triangle $\angle STV = 141°$ and $\angle TUV = 73°$, find g.

Solution

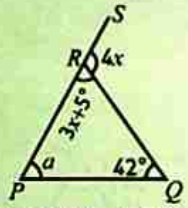
$g + 73° = 141°$

(exterior angle property of a triangle)

$g = 141° - 73°$

$g = 68°$

Example 670



In the triangle $\angle PQR = 42°$, find the value of a and x.

Solution

$4x + 3x + 5° = 80°$ (angles on a straight line)

$7x + 5° = 180°$

$7x = 180° - 5°$

$7x = 175°$

$\frac{7x}{7} = \frac{175°}{7}$

$x = 25°$

$\angle QRS = 4x = 4(25)$

$= 100°$

BEST BRAIN

$a + 42° = 100°$

(exterior angle property of a triangle)

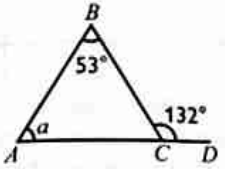
$a = 100° - 42°$

$a = 58°$

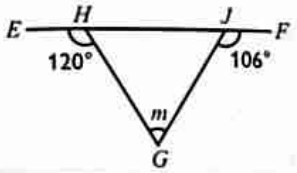
EXERCISE 192 Date

Find the values of the letters in each of the following diagrams.

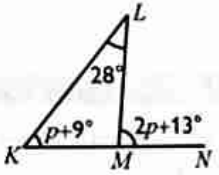
(a)



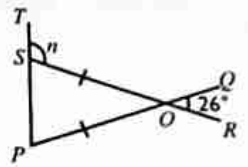
(b)



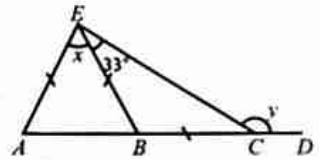
(c)



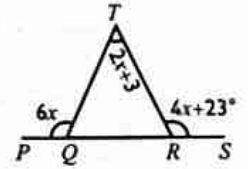
(d)



(e)



(f)



BEST BRAIN

B8.3.1.1.3 Construct and bisect angles

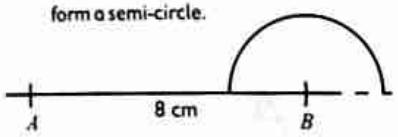
In Basic 7, we learnt how to construct angles such as 60°, 30°, 45°, 90° and 75°. In this section, we shall construct angles such as 120°, 105°, 135° and 150°.

Construction of 120°

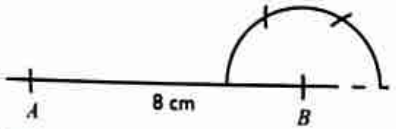
Construct $\angle ABC = 120°$ where $|AB| = 8$ cm

Step 1: Construct $|AB| = 8$ cm

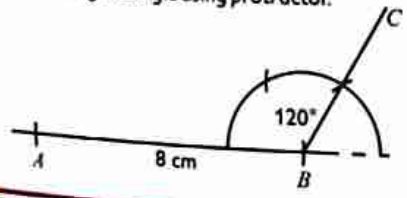
Step 2: The angle must be constructed at point B (the middle letter of the angle given). Place the metal end of the compass at B, open the compass by a reasonable radius and swing an arc from line AB to form a semi-circle.



Step 3: With the same opening in step 2, place the compass at the point where the semi-circle start on AB and swing an arc on the semi-circle. Place the metal end on the new arc and swing another arc.



Step 4: Draw a line through B and the second arc on the semi-circle and label the angle formed by AB and BC as 120°. Verify the angle using protractor.

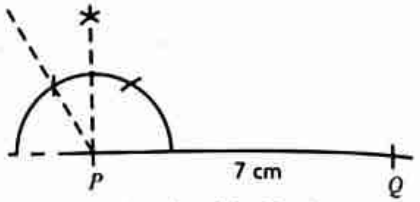


Construction of 105°

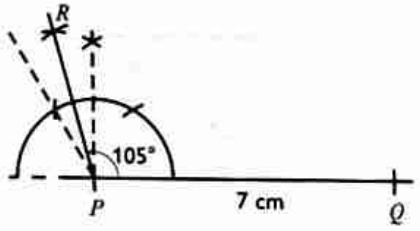
Construct $\angle QPR = 105°$ where $|PQ| = 7$ cm

Step 1: Construct $|PQ| = 7$ cm

Step 2: Construct angle 90° and 120° at P, with a broken line.



Step 3: Bisect the red and the blue dots and draw a line through the point of intersection of the arcs and P. Label the angle between QP and PR as 105°.

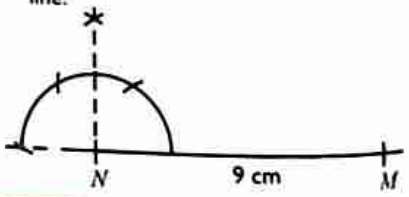


Construction of 135°

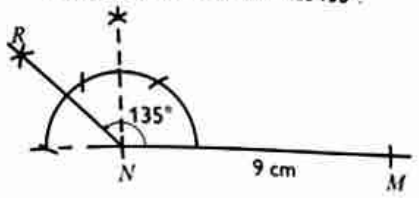
Construct $\angle MNR = 135°$ where $|MN| = 9$ cm

Step 1: Construct $|MN| = 9$ cm

Step 2: Construct angle 90° at N, with a broken line.



Step 3: Bisect the red and the green dots and draw a line through the point of intersection of the arcs and N. Label the angle between MN and NR as 135°.

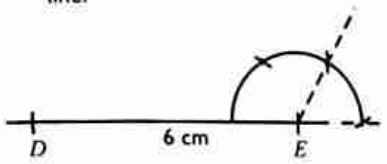


Construction of 150°

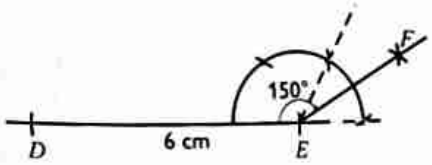
Construct $\angle DEF = 150°$ where $|DE| = 6$ cm

Step 1: Construct $|DE| = 6$ cm

Step 2: Construct angle 120° on E, with a broken line.



Step 3: Bisect the red and the green dots and draw a line through the point of intersection of the arcs and F. Label the angle between DE and EF as 150°.



EXERCISE 193 Date

Construct each of the following angles.

(a) $\angle QPR = 105°$ where $|QP| = 5$ cm

(b) $\angle KLM = 120°$ where $|LM| = 9$ cm

(c) $\angle XZY = 150°$ where $|ZY| = 7$ cm

(d) $\angle BAC = 135°$ where $|AC| = 10$ cm

B8.3.1.1.4 Construct scalene, isosceles, equilateral, obtuse and acute triangles

In this section, we shall use our knowledge on how to construct line segments and angles to construct some triangles with given information.

Example 671

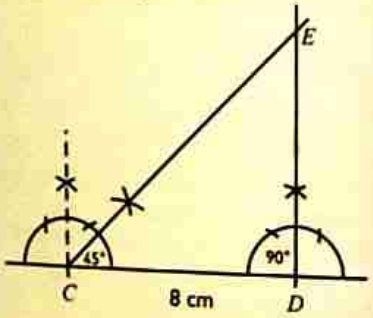
(a) Using a ruler and a pair of compasses only, construct triangle CDE in which $|CD| = 8$ cm, $\angle CDE = 90°$ and $\angle DCE = 45°$.

(b) Measure line CE.

Solution

Steps

(a) 1. Construct line segment $|CD| = 8$ cm
2. Construct angle 90° at D.
3. Draw a line from D through the point of intersection in step 2.
4. Construct angle 45° at C.
5. Draw a line from C through the point of intersection in step 4 to meet the line in step 3 at E.
6. Label the triangle using the information given in the question.



(b) $|CE| = 11.4$ cm

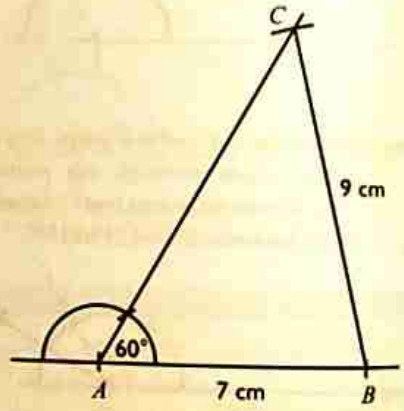
Example 672

(a) Using a ruler and a pair of compasses only, construct triangle ABC such that $\angle BAC = 60°$, $|AB| = 7$ cm and $|BC| = 9$ cm.

(b) Measure $|AC|$.

Solution

(a) 1. Construct $|AB| = 7$ cm
2. Construct angle 60° at A.
3. Draw a line from A through the point of intersection in step 2.
4. Using the pair of compasses, measure 9 cm on the rule, place the metal point at B and swing an arc above $\overline{AB}$ to intersect the line drawn in step 3 at C.
5. Label the triangle using the information given in the question.



(b) $AC = 10.1$ cm

BEST BRAIN

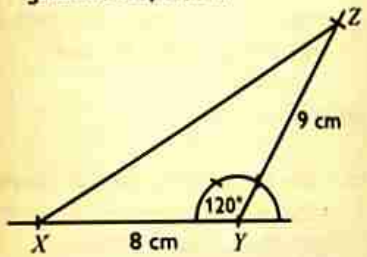
Example 673

(a) Using a ruler and a pair of compasses only, construct triangle XYZ such that $|XY| = 8$ cm, $|YZ| = 9$ cm and $\angle XYZ = 120°$.

(b) Measure XZ.

Solution

(a) 1. Construct $|XY| = 8$ cm
2. Construct angle 120° at Y.
3. Draw a line from Y through the point of intersection in step 2.
4. Using the pair of compasses, measure 9 cm on the rule, place the metal point at Y and swing an arc above XY.
5. Join a line from X to Z.
6. Label the triangle using the information given in the question.



(b) $|XZ| = 14.7$ cm

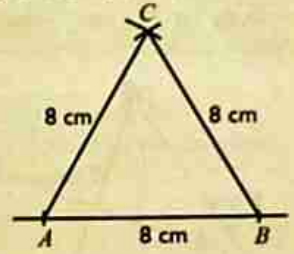
Example 674

(a) Using a ruler and a pair of compasses only, construct triangle ABC such that $|AB| = |AC| = |BC| = 8$ cm.

(b) What triangle is ABC?

Solution

(a) 1. Construct $|AB| = 8$ cm
2. Using the pair of compasses, measure 8 cm on the rule, place the metal point at A and swing an arc above $\overline{AB}$.
3. With the same radius as in step 2, place the metal point at B and swing an arc to intersect the arc in step 2 at C.
4. Label the triangle using the information given in the question.



(b) Equilateral triangle.

Example 675

(a) Using a ruler and a pair of compasses only, construct triangle PQR with $|PQ| = 7$ cm, $|PR| = 8$ cm and $|QR| = 7$ cm.

(b) (i) Construct a perpendicular bisector of $|QR|$ to meet $\overline{QR}$ at O.
(ii) Measure $|OQ|$.
(iii) Measure $\angle PQR$.

Solution

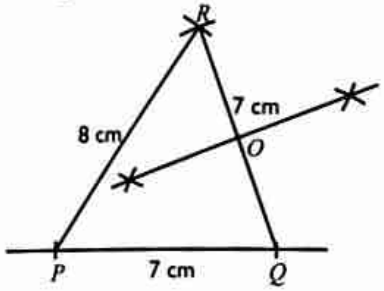
(a) 1. Construct $|PQ| = 7$ cm
2. Using the pair of compasses, measure 8 cm on the rule, place the metal point at P and swing an arc above $\overline{PQ}$.
3. Using the pair of compasses, measure 7 cm on the rule, place the metal point at Q and swing an arc to meet the arc in step 2 at R.
4. Complete the triangle by joining a line from P to R and from Q to R.
5. Label the triangle using the information given in the question.

BEST BRAIN

(b) (i) Bisect line QR and label the point of intersection O.

(ii) Use a pair of dividers to measure $\overline{OQ}$ and read from the ruler.

(iii) Place the middle point of the protractor at Q, making sure that the base line is perfectly on PQ and measure angle PQR.



(b) (ii) $|OQ| = 3.5$ cm

(iii) $\angle PQR = 70°$

EXERCISE 194 Date.....

1. (a) Using a ruler and a pair of compasses only, construct triangle MNR with $|MN| = 9$ cm, $|MR| = 6$ cm and $|NR| = 8$ cm.
   (b) What type of triangle is MNR.

2. (a) Using a ruler and a pair of compasses only, construct triangle XYZ such that $|XY| = 8$ cm, $|XZ| = |YZ| = 9$ cm.
   (b) Construct a perpendicular bisector of $\overline{XY}$.
   (c) Measure angle XZY.

3. (a) Using a ruler and a pair of compasses only, construct triangle ABC such that $|BC| = 8$ cm, $\angle ABC = 60°$ and $\angle ACB = 45°$.
   (b) Measure AB.

4. (a) Using a ruler and a pair of compasses only, construct triangle PQR such that $|QR| = 9$ cm, $\angle QRP = 75°$ and $PQ = 10$ cm.
   (b) Measure PR.

BEST BRAIN

B8.3.1.1.5 Construct loci under given conditions

In this section, we shall use construction to explain how some loci are constructed under a given conditions. We shall learn how to construct locus of points equidistant from

(a) a point;
(b) two given points;
(c) two given lines;
(d) a given line.

Constructing a locus from a point

Example 676

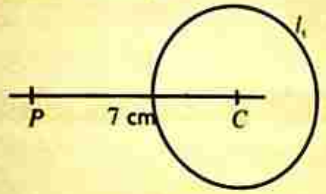
Given that $|PC| = 7$ cm, construct the locus l_1 of points 3 cm from C.

Solution

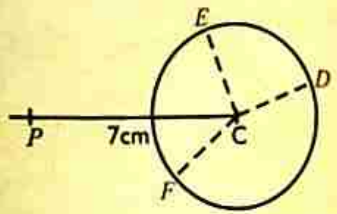
Steps

1. Construct $|PC| = 7$ cm
2. Open the compass by a radius of 3 cm, place the metal part at C and draw a circle.
3. Label the circle as l_1.

(a)



(b)



From (b), we observe that $|CD| = |CE| = |CF|$.

The locus of points equidistant from a given point with a given distance from the point is a circle.

Constructing a locus from two points

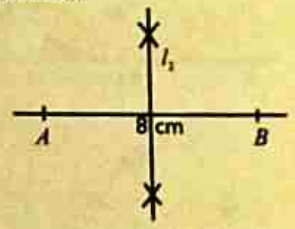
Example 677

Given that $|AB| = 8$ cm, construct the locus l_1 of points equidistant from A and B.

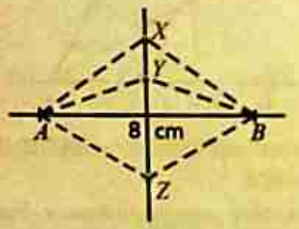
Solution

1. Construct $|AB| = 8$ cm
2. Bisect AB and draw a line through the point of intersection.

(a)



(b)



From (b), we observe that $|AY| = |BY|$, $|AX| = |BX|$ and $|AZ| = |BZ|$.

The locus of points equidistant from two points is a perpendicular bisector of the points.

Constructing a locus from two lines

Example 678

Given that $|MN| = 8$cm, $|NR| = 7$ cm and $\angle MNR = 60°$, construct the locus l_1 of points equidistant from $\overline{MN}$ and $\overline{NR}$.

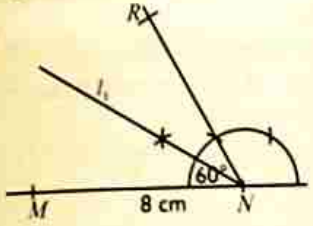
BEST BRAIN

Solution

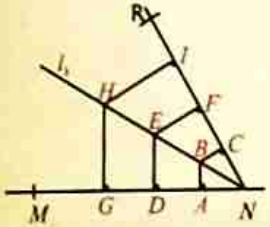
Steps

1. Construct $|MN| = 8$ cm.
2. Construct angle 60° at N.
3. Using the pair of compasses, measure 7 cm on the rule, place the metal point at N and swing an arc above $\overline{MN}$. Join the line $\overline{NR}$.

(a)



(b)



It can be observed that $|AB| = |BC|$, $|DE| = |EF|$ and $|GH| = |HI|$.

The locus of points equidistant from two intersecting lines is the angle bisector of the lines.

Constructing a locus from a line

Example 679

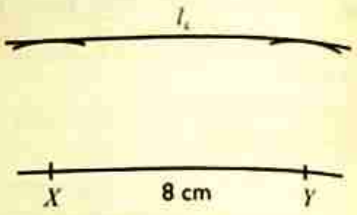
Given that $|XY| = 8$ cm, construct the locus l_4 of points 4 cm from $\overline{XY}$.

Solution

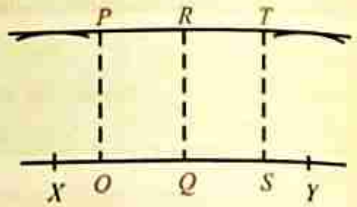
Steps

1. Construct XY= 8 cm.
2. Using the pair of compasses, measure 4 cm on the rule, place the metal point at X and swing an arc above XY. Repeat the same step at Y.
3. Join the two arcs with a straight line.

(a)



(b)



It can be observed that $|OP| = |QR| = |ST|$.

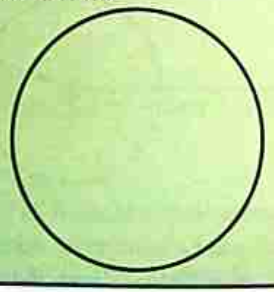
The locus of points equidistant from a line is a pair of line parallel to a given line.

We can say that a locus is a set of points equidistant from a given specified condition.

Locating the centre of a given circle

Example 680

Use geometric construction to locate the centre A of the circle below.

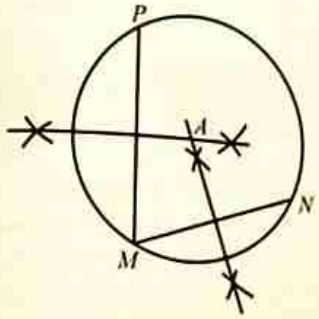


BEST BRAIN

Solution

Steps

1. Draw a chord $\overline{MN}$ from any point on the circumference to another point.
2. Draw another chord $\overline{MP}$ on the other side of $\overline{MN}$.
3. Bisect lines $\overline{MN}$ and $\overline{MP}$ respectively.
4. Locate the point of intersection of $\overline{MN}$ and $\overline{MP}$ as A.



Point A is the centre of the circle.

Constructing a regular hexagon within a circle given the length of side

Example 681

(a) Construct a hexagon $ABCDEF$ such that $|AB| = 5$ cm.

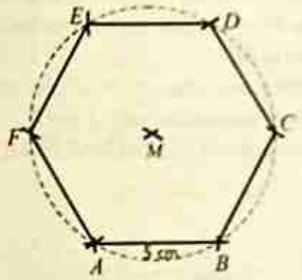
(b) If M is the centre of the hexagon, measure $\angle AMB$.

(c) Measure $\angle BMC$, $\angle CMD$, $\angle DME$, $\angle EMF$ and $\angle FMA$ and compare the angles.

Solution

Steps

1. Draw $|AB| = 5$ cm.
2. With the same radius as $\overline{AB}$, swing an arc above AB at A and at B to meet at M.
3. With M as centre and radius as $\overline{AB}$, draw a circle.
4. With the same radius as in step 3, swing an arc from B on the circle.
   Mark the point of intersection of the arc and the circle as C.
5. Continue the steps as in 4 and label the points of intersection as D, E and F.



(b) $\angle AMB = 120°$

(c) $\angle BMC = 120°$, $\angle CMD = 120°$, $\angle DME = 120°$, $\angle EMF = 120°$ and $\angle FMA = 120°$

All the angles are the same.

Using intersecting circles to construct a regular hexagon

Example 682

(a) Construct a hexagon $KLMNPR$ whose centre is O, such that $|OK| = 7$ cm

(b) Measure and compare the sides of the hexagon.

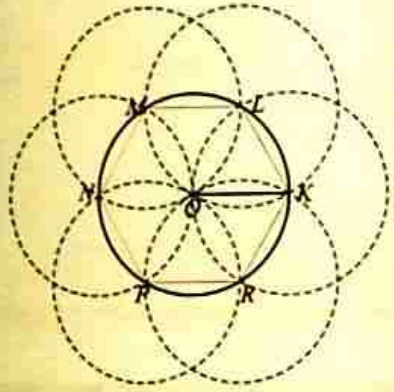
(c) Measure $\angle KOL$, $\angle LOM$, $\angle MON$, $\angle NOP$, $\angle POR$ and $\angle ROK$ and compare the angles.

BEST BRAIN

Solution

Steps

1. Use a pair of compasses to measure 7 cm. With the same radius, draw a circle
2. With the same radius, place the metal part of the compass at K and draw a circle to intersect the first circle at L.
3. With the same radius as in step 3, place the metal part of the compass at L and draw a circle. Mark the point of intersection on the circles as M.
4. Continue the steps as in 4 and label the points of intersection as N, P and R
5. Join the points $\overline{KL}, \overline{LM}, \overline{MN}, \overline{NP}, \overline{PR}$ and $\overline{RK}$.

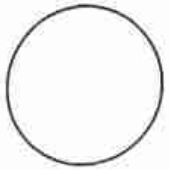
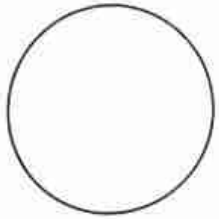


(b) $|KL| = 7$ cm, $|LM| = 7$ cm, $|MN| = 7$ cm, $|NP| = 7$ cm, $|PR| = 7$ cm and $|RK| = 7$ cm
The length of sides of the hexagon are all the same.

(c) $\angle KOL = 60°, \angle LOM = 60°$, $\angle MON = 60°, \angle NOP = 60°$, $\angle POR = 60°$ and $\angle ROK = 60°$
All the angles are the same.

EXERCISE 195 Date

1. Given that $|AB| = 10$ cm, construct the locus
   (a) l_1 of points 3 cm from $\overline{AB}$;
   (b) l_2 of points 7 cm from $\overline{AB}$.

2. Use geometric construction to locate the centre of the circles below.
   (a)
   
   (b)
   

3. (a) Construct a hexagon $PQRSTU$ such that $|PQ| = 4$ cm.
   (b) Given that O is the centre of the hexagon, measure $\angle POQ$.
   (c) Measure $\angle QOR$, $\angle ROS$, $\angle SOT$, $\angle TOU$ and $\angle UOP$ and compare the angles

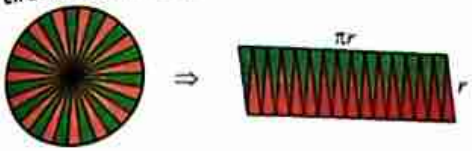
4. (a) Construct a hexagon $ABCDEF$ whose centre is O, such that $\overline{OA} = 6$ cm.
   (b) Measure and compare the sides of the hexagon.
   (c) Measure $\angle AOB, \angle BOC, \angle COD, \angle DOE, \angle EDF$ and $\angle FOA$ and compare the angles.

BEST BRAIN

STRAND 3.2 MEASUREMENT

B8.3.2.1.1 Deduce the formula for the area of a circle and solve problems on it.

Consider the circle with radius r and circumference $2r\pi$.



When we divide the circle into smaller sectors and arrange them as shown, we obtain a rectangle with length πr and breadth r.

But area of a rectangle = length × breadth

$= \pi r \times r$

$= \pi r^2$

Therefore Area of a circle $= \pi r^2$

But $2r = d$ (diameter)

$r = \frac{d}{2}$

Area of a circle $= \pi \left(\frac{d}{2}\right)^2$

$= \pi \left(\frac{d^2}{4}\right)$

$= \frac{1}{4}\pi d^2$

Example 683

The base of a milo container is circular with radius 7 cm. Find the area of the base of the milo container.

[Take $\pi = \frac{22}{7}$]

Solution

Area of a circle $= \pi r^2$

$= \frac{22}{7} \times 7^1$

$= \frac{22}{7} \times 7 \times 7$

$= 22 \times 7$

$= 154$ cm^2

Example 684

A dining table has a circular surface. Given that the diameter of the table is 42 cm, find the area of the top of the table.

[Take $\pi = \frac{22}{7}$]

Solution

Area of the circular table $= \frac{1}{4}\pi d^2$

$= \frac{1}{4} \times \frac{22}{7} \times 42^2$

$= \frac{1}{4} \times \frac{22}{7} \times 42 \times 42$

$= \frac{1}{2} \times 11 \times 42$

$= 33 \times 42$

$= 1{,}386$ cm^2

Example 685

Mr. Ampah has a piece of land in the form of a circle. If the perimeter of the circular land is 880 m, find the
(a) radius of the land;
(b) area of the land.

[Take $\pi = \frac{22}{7}$]

BEST BRAIN

Solution

(a) We know that perimeter of a circle is the same as the circumference.

Circumference of a circle = $2\pi r$

$880 = 2 \times \frac{22}{7} \times r$

$880 = \frac{44r}{7}$ (multiply both sides by 7)

$880 \times 7 = \frac{44r}{7} \times 7$

$880 \times 7 = 44r$ (divide both sides by 44)

$\frac{880 \times 7}{44} = \frac{44r}{44}$

$20 \times 7 = r$

$140 = r$

(b) Area of the circle = πr^2

$= \frac{22}{7} \times 140^2$

$= \frac{22}{7} \times 140 \times 140$

$= 22 \times 2800$

$= 61{,}600\ m^2$

Example 686

Mrs. Ayivor has a circular swimming pool in her house. If the area of the pool is 154 m, find the

(a) diameter;

(b) perimeter of the pool.

[Take $\pi = \frac{22}{7}$]

Solution

(a) Area of the circular pool = πr^2

$154 = 2 \times \frac{22}{7} \times r^2$ (multiply both sides by 7)

$154 \times 7 = \frac{22}{7} \times r^2 \times 7$

$154 \times 7 = 22r^2$

$\frac{154 \times 7}{22} = \frac{22r^2}{22}$

$7 \times 7 = r \times r$

$7 = r$

Diameter = $2r$

$= 2 \times 7$

$= 14$ m

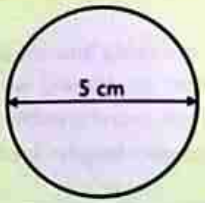
(b) Perimeter or circumference of the pool = $2\pi r$

$= 2 \times \frac{22}{7} \times 7$

$= 44$ m

Example 687

Find the area of the circle below to 1 decimal place.



Solution

The radius is half of the diameter, so it is half of 5 = 2.5 cm.

Next we need to square the radius:

$2.5^2 = 2.5 \times 2.5 = 6.25\ cm^2$.

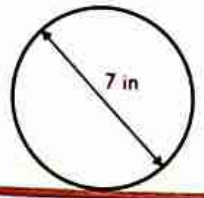
Finally, we need to multiply this amount by π.

So the area of this circle is $\pi \times r^2 = \pi \times 6.25\ cm^2$.

If we take pi = 3.14, it gives us an area of $3.14 \times 6.25 = 19.6$ square cm or $19.6\ cm^2$ to 1 decimal place.

Example 688

Find the area of the circle below to the nearest inch.



BEST BRAIN

The radius of the circle is half of the diameter which is half of 7 = 3.5 inches.

Next we need to square the radius:

$3.5^2 = 3.5 \times 3.5 = 12.25$ cm.

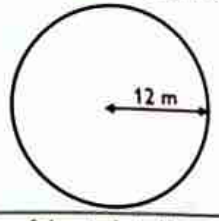
Finally we need to multiply this amount by pi (π).

So the area of this circle is $\pi \times r^2 = \pi \times 12.25$ cm.

If we take pi = 3.14, this gives us an area of $3.14 \times 12.25 = 38.5$ square inches or $38.5\ in^2$ to 1 decimal place.

Example 689

Find the area of the circle below and give your answer to the nearest whole number.



The radius of the circle is 12 m, so the square of the radius is $12^2 = 12 \times 12 = 144$.

Finally we need to multiply this amount by pi (π).

So the area of this circle is $\pi \times r^2 = \pi \times 144$ m.

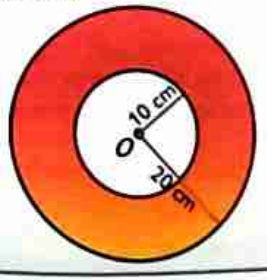
This gives us an area of $3.14 \times 144 = 452$ square metre or $452\ m^2$ to the nearest whole number.

Example 690

In the figure given below two circles with radii 20 cm and 10 cm have the same centre.

Find the area of the shaded portion.

Take $\pi = 3.14$



Solution

First find the area of the outer and inner circles.

Radius of inner circle = 10 cm

Radius of outer circle = 20 cm

Area of inner circle = 3.14×10^2 = 314 cm^2

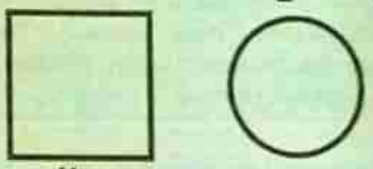
Area of outer circle = 3.14×20^2 = 1,256 cm^2

Therefore, area of the shaded portion is $1256 - 314 = 942\ cm^2$

Example 691

A wire is bent in the form of a square of side 44 cm. If the same wire is bent in the form of a circle, find the area of the circle thus formed.

[Take $\pi = \frac{22}{7}$]



Solution

Perimeter of the square

= 4 x side = 4 x 44 = 176 cm

Since the perimeter of the square = Circumference of the circle formed.

Circumference of a circle = $2\pi r$

$= 2 \times \frac{22}{7} \times r = 176 \Rightarrow \frac{44}{7} r = 176$

$44r = 1232 \Rightarrow r = 28$

Now, area of the circle = $\frac{22}{7} \times 28^2$

$\frac{22}{7} \times 28 \times 28$

Therefore, area of the circle = 2464 cm^2

BEST BRAIN

Example 692

A race track is in the form of a circular ring. The inner radius of the track is 58 yd and the outer radius is 63 yd. Find the area of the race track.

Solution

Given: $R = 63$ yd, $r = 56$ yd

Let the area of the outer circle be A_1 and the area of the inner circle be A_2

Area of race track $= A_1 - A_2$

$= \pi R^2 - \pi r^2 = \pi(63^2 - 56^2) = \frac{22}{7} \times 833$

$= 2,618$ square yards.

The area of the race track is 2618 square yards.

Example 693

A wire is in the shape of an equilateral triangle. Each side of the triangle measures 7 in. The wire is bent into the shape of a circle. Find the area of the circle that is formed.

Solution

Perimeter of the equilateral triangle

$= 3 \times$ side $= 3 \times 7 = 21$ inches.

Since the perimeter of the equilateral triangle = Circumference of the circle formed

Thus, the perimeter of the triangle is 21 inches.

Circumference of a circle

$= 2\pi r = 2 \times \frac{22}{7} \times r = 21$

$r = \frac{21 \times 7}{44} = 3.34$

Therefore, the radius of the circle is 3.34 cm.

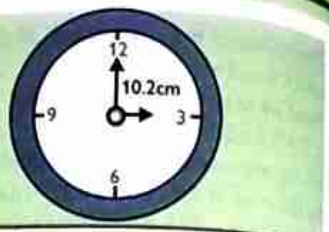
Area of a circle $= \pi r^2 = \frac{22}{7} \times (3.34)^2$

$= 35.042$ square inches

The area of a circle is 35.042 square inches.

Example 694

Find the area swept by a clock's minute hand in one hour. The length of the minutes hand is 10.2cm. What is the area swept in 30 minutes and also in 2 hours by the same hand?



Solution

The minute hand sweeps one full circle in one hour. So we need to find the area of a circle with radius equal to the length of the minute hand. Then if we divide the result by 2, we will get the area swept in 30 minutes.

Also we will multiply the one hour result by 2 to get the area swept by the same hand in 2 hours.

Now, radius of the circle = length of the minute hand = 10.2 cm

Area swept in 1 hour

= area swept in full circle $= \pi r^2$.

Hence, the area swept in 1 hour

$= 3.14 \times 10.2 \times 10.2$

$= 326.69$ cm^2

Area swept in 30 minutes is $\frac{326.69}{2} = 163.34$ cm

Also, area swept in 2 hours is 326.69×2

$= 653.37$ cm^2

Example 695

The time shown in a circular clock is 3:00 pm. The length of the minute hand is 21 units. Find the distance travelled by the tip of the minute hand when the time is 3:30 pm.

Solution

When the minute hand is at 3:30 pm, it covers half of the circle. So, the distance travelled by the minute hand is actually half of the circumference.

Distance $= \pi r$

(where r is the length of the minute hand).

Hence the distance covered

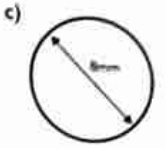
$= \frac{22}{7} \times 21 = 22 \times 3 = 66$ units

BEST BRAIN

EXERCISE 196 Date

1. A circular football field has a radius of 35 m. Find the area of the field. [Take $\pi = \frac{22}{7}$]
2. A barrel has a circular base with diameter of 84 cm. Find the base area of the barrel. [Take $\pi = \frac{22}{7}$]
3. Mr Ankrah's dining hall is circular with radius 1.4 m. Find the
   (a) perimeter of the dining hall;
   (b) area of the dining hall. [Take $\pi = \frac{22}{7}$]
4. Mrs Yeboah has a backyard garden. The garden is in the form of a circle with perimeter 132 m. Find the
   (a) radius of the garden;
   (b) area of the garden.
5. The area of a circular table is 22464 cm, find the
   (a) diameter of the table,
   (b) perimeter of the table.

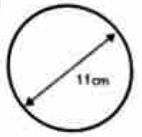
[Take $\pi = \frac{22}{7}$]

c) 

A = ...

d)

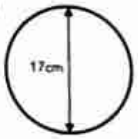
A = ...

e) 

A = ...

f)

A = ...

g) 

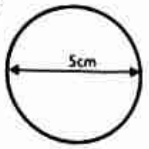
A = ...

h)

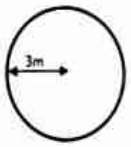
A = ...

EXERCISE 197 Date

1. Work out the area of these circles using the diameter and radius measurements. Give your answers to 1 dp.

a) 

A = ...

b) 

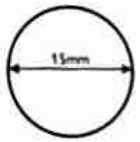
A = ...

i)

A = ...

j)

A = ...

k) 

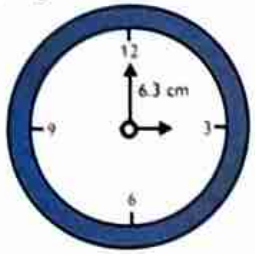
A = ...

l)

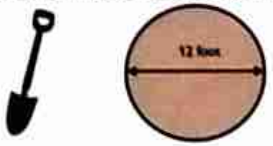
A = ...

BEST BRAIN

2. Find the area swept by a clock's minute hand in one hour. The length of the minutes hand is 6.3 cm. What is the area swept in 25 minutes by the same hand?



3. Akomeah digs a 12-foot wide circular hole in his garden. What is the area of his hole?

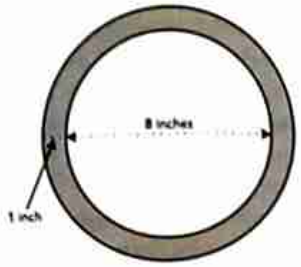


4. A tin of beans has a diameter of 3 inches. What is the area of the top of the tin?

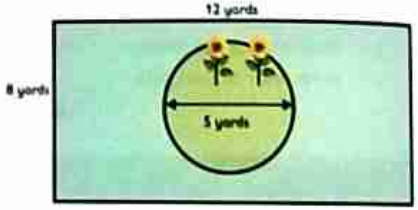
5. A cell phone mast has a range of 5 miles in all directions. Work out the area covered by one mast.

6. A circular mirror has an area of 144π square inches. What is the diameter of the mirror?

7. A circular mirror has a 1 inch frame around it. The mirror itself is 8 inches in diameter. What is the area of the frame?

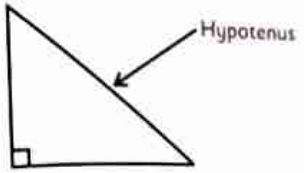


8. A circular flower bed 5 yards in diameter is created in the middle of a rectangular lawn area measuring 12 yards by 8 yards. What is the area of the lawn that is left?



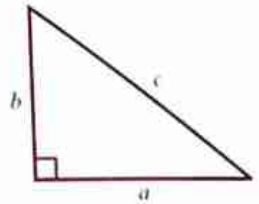
B8.3.2.1.2&3 Show relationship between hypotenuse and the other two sides of a right angled triangle

In this section, we shall deduce the relationship between the hypotenuse (the side opposite to the right angle) and the other two sides of a right angled triangle. Look at a right angled triangle.

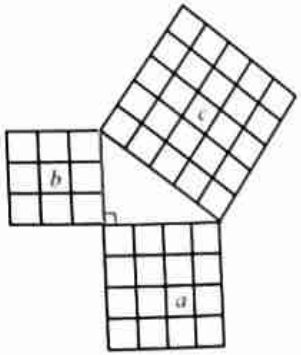


Let us use square boxes to determine the area of each square.

(1)



(2)



In the diagram, in (2), a square has an area of $1\ m^2$.

Area $a = 9 = 3^2$

Area $b = 16 = 4^2$

Area $c = 25 = 5^2$

We can observe that area a + area b = area c

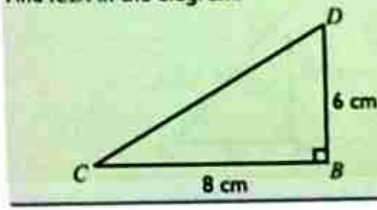
We can therefore write an algebraic sentence from (1) as $c^2 = a^2 + b^2$.

This relationship from a right angled triangle is called the Pythagoras theorem.

The theorem states that the sum of the squares of the opposite and the adjacent of a right-angled triangle is the same as the square of the hypotenuse.

Example 696

Find $|CD|$ in the diagram.



Solution

$|CD|^2 = |BC|^2 + |BD|^2$

$|CD|^2 = 8^2 + 6^2$

$|CD|^2 = 64 + 36$

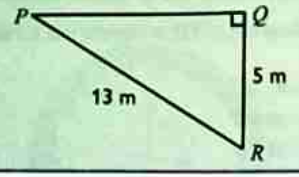
$|CD|^2 = 100$

$|CD|^2 = 10^2$

$|CD| = 10$ cm

Example 697

Find $|PQ|$ in the diagram.



Solution

$|PR|^2 = |PQ|^2 + |QR|^2$

$13^2 = |PQ|^2 + 5^2$

$169 = |PQ|^2 + 25$

$169 - 25 = |PQ|^2$

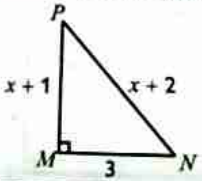
$144 = |PQ|^2$

$12^2 = |PQ|^2$

$12 \text{ m} = |PQ|$

Example 698

Find the value of x in the diagram.



Solution

$|PN|^2 = |MN|^2 + |MP|^2$

$(x+2)^2 = 3^2 + (x+1)^2$

$(x+2)(x+2) = 9 + (x+1)(x+1)$

$x(x+2) + 2(x+2) = 9 + x(x+1) + 1(x+1)$

$x^2 + 2x + 2x + 4 = 9 + x^2 + x + x + 1$

$x^2 + 4x + 4 = 9 + x^2 + 2x + 1$

(group all variables on one side)

$x^2 - x^2 + 4x - 2x = 9 + 1 - 4$

$2x = 6$

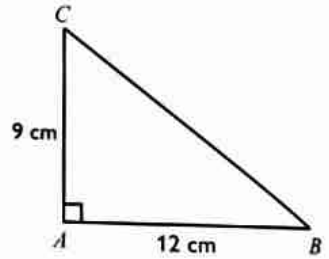
$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

EXERCISE 198 Date...

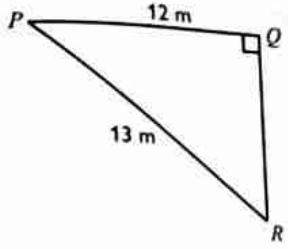
1. Find $|CB|$, $|QR|$ and $|DF|$ in the following diagrams.

(a)

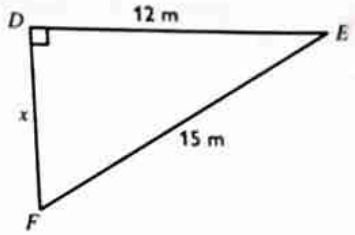


BEST BRAIN

(b)

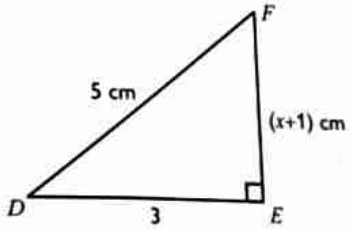


(c)

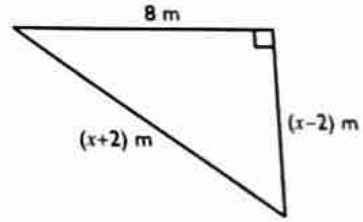


2. Find the value of x in each of the diagrams.

(a)



(b)



BEST BRAIN

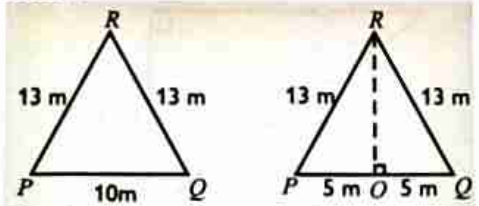
B8.3.2.1.4 Use Pythagoras theorem to solve real life problems

There are many real life problems that use the concept of Pythagoras theorem to solve. In this section, we shall use our knowledge on Pythagoras theorem to solve some of such problems.

Example 699

An isosceles triangle has length of sides $|PQ| = 10$ m, $|PR| = 13$ m and $|QR| = 13$ m. Find the area of the triangle.

Solution



Using Pythagoras theorem to find the perpendicular height $|OR|$

$13^2 = 5^2 + |OR|^2$

$169 = 25 + |OR|^2$

$169 - 25 = |OR|^2$

$144 = |OR|^2$

$12^2 = |OR|^2$

$12\text{ m} = |OR|$

Area of the triangle $= \frac{1}{2} \times |PQ| \times |OR|$

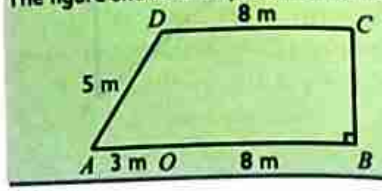
$= \frac{1}{2} \times 10 \times 12$

$= 5 \times 12$

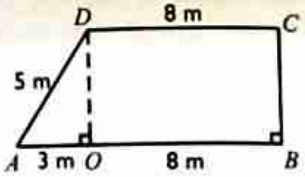
$= 60\text{ m}^2$

Example 700

The figure shows a trapezium. Find the area.



Solution



Area of the trapezium = area of ΔOAD + area of the rectangle $OBCD$

Let us use Pythagoras theorem to find the perpendicular height $|OD|$.

$5^2 = 3^2 + |OD|^2$

$25 = 9 + |OD|^2$

$25 - 9 = |OD|^2$

$16 = |OD|^2$

$4^2 = |OD|^2$

$4\text{ m} = |OD|$

Area of $\Delta OAD = \frac{1}{2} \times 3 \times 4$

$= 3 \times 2$

$= 6\text{ m}^2$

Area of the rectangle $OBCD = 8 \times 4$

$= 32\text{ m}^2$

Area of the trapezium $= 6\text{ m}^2 + 32\text{ m}^2$

$= 38\text{ m}^2$

BEST BRAIN

Example 701

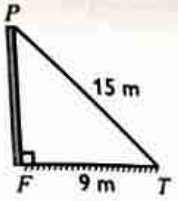
A ladder of length 15 m leans against a vertical wall. The distance between the foot (F) of the wall and the foot (T) of the ladder is 9 m.

(a) Represent the information on a diagram.

(b) Find the distance between the foot and the point where the ladder touches the wall.

Solution

(a)



(b) Using Pythagoras theorem to find the perpendicular distance between the foot and the point where the ladder touches the wall FP.

$15^2 = 9^2 + |FP|^2$

$225 = 81 + |FP|^2$

$225 - 81 = |FP|^2$

$144 = |FP|^2$

$12^2 = |FP|^2$

$12\text{ m} = |FP|$

Let us use our knowledge on the Pythagoras theorem concept to solve some problems that we encounter in daily life.

Example 702

The diagram below shows the roof of a house. Suppose you need to replace a beam (where the question mark is) that connects the two sides of the roof. How long should the beam be?

Solution

The beam is the horizontal line and its length is shown with a red line. Notice that the sides of the roof have the same length. Furthermore, since the two sides of the roof make a right triangle, we can use the Pythagorean Theorem to find the length of the beam.

$c^2 = a^2 + b^2$

$c^2 = 25^2 + 25^2$

$c^2 = 625 + 625$

$c^2 = 1250$

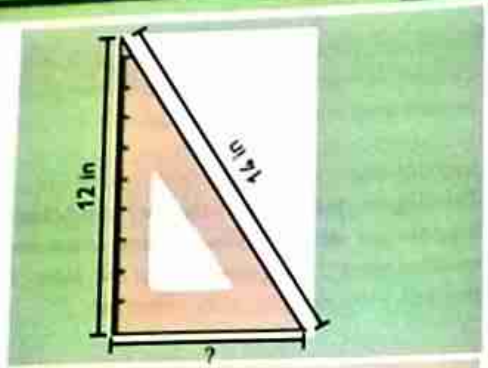
$c = \sqrt{1250} = 35.35$

The length of the beam is 35.35 feet.

Example 703

Anasewaa develops a blueprint with the help of a triangle. The longest side of the instrument measures 14 inches and one of the sides measures 12 inches. Determine the length of the other side of the instrument if the instrument has a right angle.

BEST BRAIN



Solution

$c^2 = a^2 + b^2$

$14^2 = 12^2 + b^2$

$196 = 144 + b^2$

$b^2 = 196 - 144$

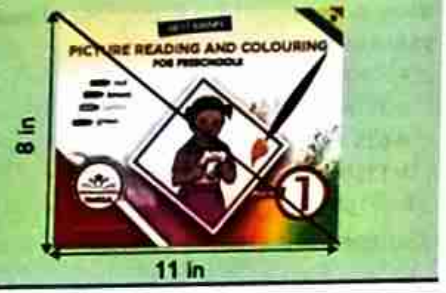
$b^2 = 52$

$b = \sqrt{52} = 7.2$

The length of the other side of the instrument is 7.2 inches.

Example 704

Carol bought a colouring book. It is 11 inches long and 8 inches wide. What is the diagonal length of the book?



Solution

$c^2 = a^2 + b^2$

$c^2 = 11^2 + 8^2$

$c^2 = 121 + 64$

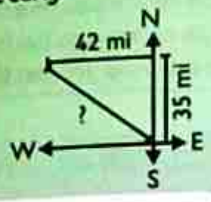
$c^2 = 185$

$c = \sqrt{185} = 13.6$

The diagonal length of the book is 13.6 inches

Example 705

Mr Wontumi is on his way home from work. He drives 35 miles due North and then 42 miles due West. Find the shortest distance he can cover to reach home early.



Solution

$c^2 = a^2 + b^2$

$c^2 = 35^2 + 42^2$

$c^2 = 1225 + 1764$

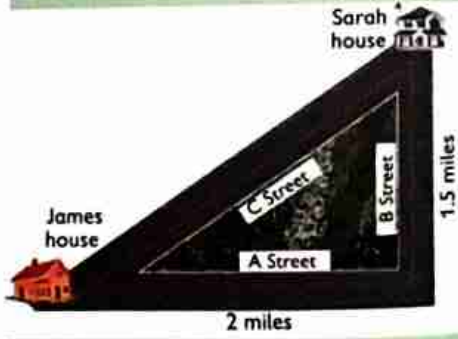
$c^2 = 2989$

$c = \sqrt{2989} = 54.7$

The shortest distance he can cover to reach home early will be 54.7 miles.

Example 706

There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to take A street and then B street. How much shorter is the direct path along C street? (Using the figure below).



Solution

By choosing the C street, he has to cover the distance,

$= \sqrt{2^2 + 1.5^2}$

$= \sqrt{(4 + 2.25)}$

$= \sqrt{6.25}$

$= 2.5$ miles

By choosing the alternative way, he has to cover the distance $= 2 + 1.5 = 3.5$ miles

The difference between these two paths

$= 3.5 - 2.5 = 1$ mile

So, by choosing the direct path, he may save 1 miles faster than other way.

Quick Tip!

One way of making quick calculations of the Pythagoras Theorem is by knowing the Pythagoras triples.

What are Pythagorean Triples?

The integer solutions to the Pythagorean Theorem, $a^2 + b^2 = c^2$ are called Pythagorean Triples which contains three positive integers a, b, and c.

Example: (3, 4, 5)

By evaluating we get:

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Hence, 3, 4 and 5 are Pythagorean triples.

You can say "triplets," but "triples" is the favoured term.

(3, 4, 5)	(5, 12, 13)	(8, 15, 17)	(7, 24, 25)
(20, 21, 29)	(12, 35, 37)	(9, 40, 41)	(28, 45, 53)
(11, 60, 61)	(16, 63, 65)	(33, 56, 65)	(48, 55, 73)
(13, 84, 85)	(36, 77, 85)	(39, 80, 89)	(65, 72, 97)

Pythagoras Triples Formula

If a triangle has one angle which is a right-angle (i.e. 90°), there exists a relationship between the three sides of the triangle.

If the longest side (the hypotenuse) is r and the other two sides (next to the right angle) is called p and q, then:

$p^2 + q^2 = r^2$.

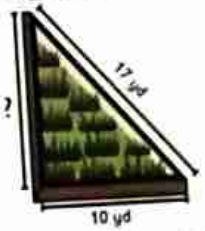
or,

The sum of the squares of the other two sides is the same as the square of the longest side.

Pythagoras triples are endless, the table above only contains the ones that are integers below 100.

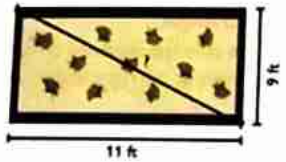
EXERCISE 199 Date

1. Ashley has a right triangular garden in her backyard. The longest side of the garden measures 17 yards. One of the sides is 10 yards long. Find the length of the other side of the garden.

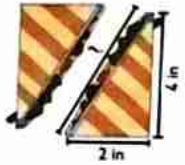


2. Mr. Richard owns an orchard that has a rectangular fence. The orchard is 36 yards long and 18 yards wide. If he walks across the diagonal length of the orchard, how much distance would he cover?

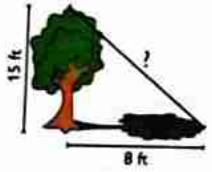
3. Find the length of the diagonal of the figure below.



4. Joey made a sandwich that was 2 inches long and 4 inches high. If he cuts the sandwich in half as shown in the figure, what would be the diagonal length of the sandwich?

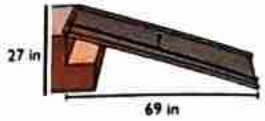


5. A 15 feet tree casts a shadow that is 8 feet long. What is the distance from the tip of the tree to the tip of its shadow?



6. The slope of the rafter is 15 feet. Half the run of the rafter measures 12 feet. Find the height of the ridge from the base.

7. A ramp is elevated to a height of 27 inches from the base. The horizontal distance between the lower end of the ramp and the base is 69 inches. What is the length of the ramp?

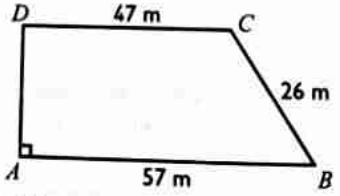


EXERCISE 200 Date

1. A garden is in the form of an isosceles triangle with sides $|AC| = 16$ m and $|AB| = |BC| = 17$ m.
Find the area of the garden.

2. A rectangular plot $PQRS$ has a perimeter of 54 m. The length of the rectangle is 7 m more than the width. If the width is x, find the
(a) value of x;
(b) area of the rectangular plot;
(c) length of $\overline{PR}$.

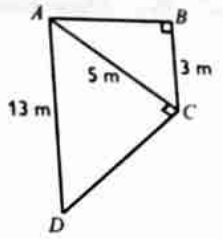
3. The figure below shows a swimming pool in the form of a trapezium.



Find the area of the swimming pool.

4. A painter placed a ladder against a vertical building to enable him paint the building. The distance between the foot of the ladder and the building is 8 m and the distance between the top of the ladder and the foot of the building is 15 m.
(a) Represent the information on a diagram,
(b) Calculate the length of the ladder.

5. The diagram shows a quadrilateral $ABCD$.

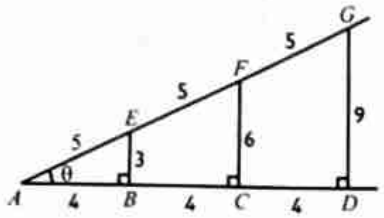


Find (a) $|AB|$
(b) $|DC|$
(c) the area of $ABCD$

6. Anokye left his house (A) and walked directly east to a junction (J) 3 km away. He then walked directly north to the church 4 km away. Calculate the distance between Anokye's house and the church.

B8.3.2.1.5 Show relationship between the basic trigonometric ratios

Consider the diagram.



Let us find the ratio of EB to AE, FC to AF and GD to AG.

$$|EB| : |AE| = \frac{3}{5}$$

$$|FC| : |AF| = \frac{6}{10} = \frac{3}{5}$$

$$|GD| : |AG| = \frac{9}{15} = \frac{3}{5}$$

We can observe that the ratios are the same. This ratio tells how wide or close the angle between AB and AE is.

The ratio is defined as sin of $\angle A$.

From the diagram, $\sin A = \frac{|EB|}{|AE|} = \frac{|FC|}{|AF|} = \frac{|GD|}{|AG|}$

Let us find the ratio of AB to AE, AC to AF and AD to AG.

$$|AB| : |AE| = \frac{4}{5}$$

$$|AC| : |AF| = \frac{8}{10} = \frac{4}{5}$$

$$|AD| : |AG| = \frac{12}{15} = \frac{4}{5}$$

We can observe that the ratios are the same. This ratio tells how wide or close the angle between AB and AE is.

The ratio is defined as cos of $\angle A$.

From the diagram, $\cos A = \frac{|AB|}{|AE|} = \frac{|AC|}{|AF|} = \frac{|AD|}{|AG|}$

Let us find the ratio of EB to AB, FC to AC and GD to AD.

$$|EB| : |AB| = \frac{3}{4}$$

$$|FC| : |AC| = \frac{6}{8} = \frac{3}{4}$$

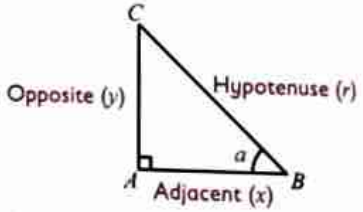
$$|GD| : |AD| = \frac{9}{12} = \frac{3}{4}$$

We can observe that the ratios are the same. This ratio tells how wide or close the angle between AB and AE is.

The ratio is defined as tan of $\angle A$.

From the diagram, $\tan A = \frac{|EB|}{|AB|} = \frac{|FC|}{|AC|} = \frac{|GD|}{|AD|}$

All the three ratios give the measure of an angle in a right angled triangle. Let us use the diagram (right angled triangle) to write the general rule for the ratios.



From the previous explanations on the ratios, we have

$$\sin(a) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r} \longrightarrow \text{SOH}$$

$$\cos(a) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r} \longrightarrow \text{CAH}$$

$$\tan(a) = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} \longrightarrow \text{TOA}$$

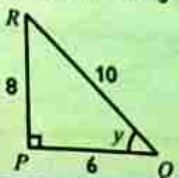
BEST BRAIN

These ratios are easily remembered by writing SOH – CAH – TOA.

It is important to note that the angle is always formed between the hypotenuse and the adjacent.

Example 707

Find $\sin(y)$ and $\tan(y)$ in the diagram below.



Solution

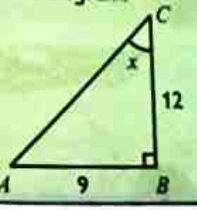
From the diagram, opposite = 8, adjacent = 6 and hypotenuse = 10

$$\sin(y) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$$

$$\tan(y) = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{6} = \frac{4}{3}$$

Example 708

Find $\cos(x)$ in the diagram.



Solution

From the diagram, opposite = 9, adjacent = 12 and hypotenuse = $|AC|$

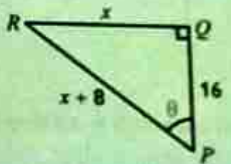
$$\cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{|AC|}$$

Using Pythagoras theorem to find $|AC|$.

$$|AC|^2 = 12^2 + 9^2$$
$$|AC|^2 = 144 + 81$$
$$|AC|^2 = 225$$
$$|AC|^2 = 15^2$$
$$|AC| = 15$$
$$\cos(x) = \frac{12}{15}$$
$$= \frac{4}{5}$$

Example 709

Find $\tan(\theta)$ in the diagram.



Solution

From the diagram, opposite = x, adjacent = 16 and hypotenuse = $x + 8$

Using Pythagoras theorem to find the value of x,

$$(x + 8)^2 = x^2 + 16^2$$
$$(x + 8)(x + 8) = x^2 + 256$$
$$x(x + 8) + 8(x + 8) = x^2 + 256$$
$$x^2 + 8x + 8x + 64 = x^2 + 256$$
$$x^2 + 16x + 64 = x^2 + 256$$

(group all the variables on one side)

$$x^2 - x^2 + 16x = 256 - 64$$
$$16x = 192$$
$$\frac{16x}{16} = \frac{192}{16}$$
$$x = 12$$

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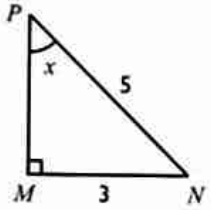
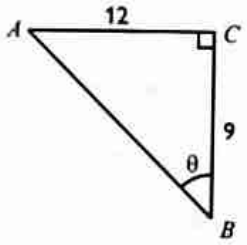
$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{16} = \frac{12}{16} = \frac{3}{4}$$

EXERCISE 201 Date.....

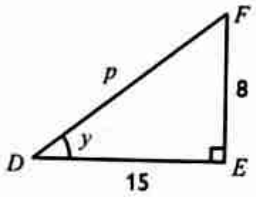
1. Find $\tan(x)$ and $\cos(x)$ in the diagram.



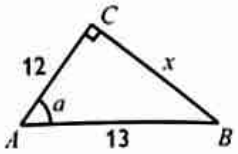
2. Find $\sin(\theta)$ and $\cos(\theta)$ in the diagram.



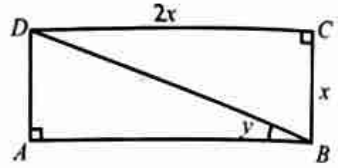
3. Find the value of p, $\sin(y)$ and $\cos(y)$ in the diagram.



4. Find $\sin(a)$ and $\tan(a)$ in the diagram.



5. The figure shows a rectangular plot whose perimeter is 54 m.



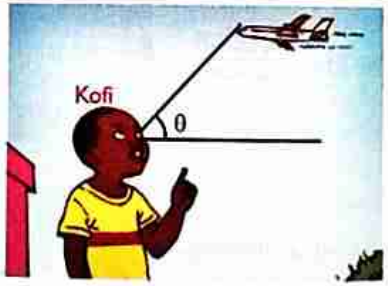
Find the value of

(a) x;

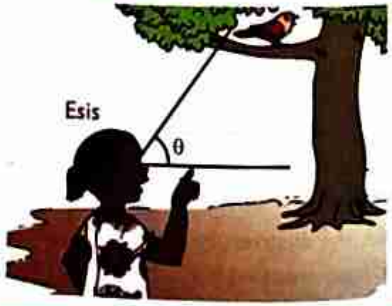
(b) $\tan(y)$.

Angles of elevation and depression

(a)



(b)



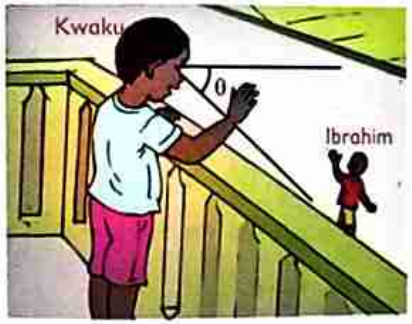
BEST BRAIN

From diagram (a), we observe that in order for Kofi to see the aeroplane, he has to look up.
From diagram (b), we observe that Esi can only see the bird on the tree if she raises the eye up.

From the two diagrams, Kofi and Esi are looking at objects above them, the angle in which they have to raise up the eye to see the objects is called elevation

(c)

(d)



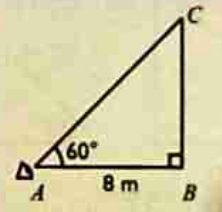
From diagram (c), Abena is above and the car is below her, in order for Abena to see the car, she has to lower the eye.
From diagram (d), the position of Kuuku is higher than Ibrahim. In order for Kuuku to see Ibrahim, he has to lower the eye at the same time Ibrahim can also see Kuuku by raising the eye.

From the diagrams (c) and (d), the angle in which Abena and Kuuku have to lower their eye to see the objects below them is called depression.
It can also be observed from diagram (d) that the angle of depression with respect to Kuuku is the same as angle of elevation with respect to Ibrahim.

Example 710

A boy observes a bird on an electric pole, the distance between the boy and the foot of the pole is 8m and the angle of elevation of the top of the pole from the boy is 60°. Find the distance between the boy and the bird, if $\cos(60) = \frac{1}{2}$

Solution



Let $|AC| = y$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenus}} = \frac{|AB|}{|AC|}$$

$$\cos(60) = \frac{|AB|}{y}$$

$$\frac{1}{2} = \frac{8}{y} \quad \text{(multiply both sides by 2)}$$

$$2 \times \frac{1}{2} = 2 \times \frac{8}{y}$$

$$1 = \frac{16}{y} \quad \text{(multiply both sides by } y\text{)}$$

$$y \times 1 = y \times \frac{16}{y}$$

$$y = 16 \text{ m}$$

Therefore the distance between the boy and the bird is 16 m.

BEST BRAIN

Example 711

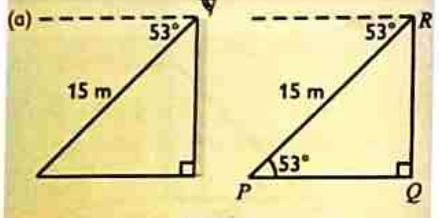
A boy on top of a building observes a goat on the same horizontal level as the foot of the building. If the distance between the boy and the goat is 15 m and the angle of depression of the goat from the boy is 53°.

(a) Draw a diagram to represent the information.

(b) Find the
(i) height of the building;
(ii) distance between the goat and the foot of the building. [Take sin (53) = $\frac{4}{5}$]

Solution

(a)



(b) (i) $\sin(53) = \frac{\text{opposite}}{\text{hypotenuse}}$

$\frac{4}{5} = \frac{|QR|}{15}$ (multiply both sides by 15)

$15 \times \frac{4}{5} = 15 \times \frac{|QR|}{15}$

$3 \times 4 = |QR|$

$12\text{ m} = |QR|$

Therefore the height of the building is 12 m

(ii) Using Pythagoras theorem

$15^2 = 12^2 + |PQ|^2$

$225 = 144 + |PQ|^2$

$225 - 144 = |PQ|^2$

$81 = |PQ|^2$

$9^2 = |PQ|^2$

$9\text{ m} = |PQ|$

Therefore the distance between the goat and the foot of the building is 9 m.

Example 712

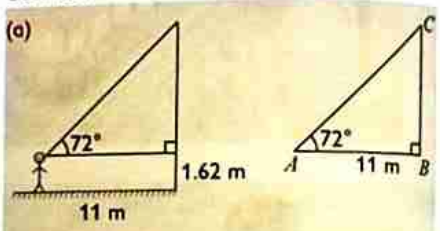
A hunter of height 1.62 m observes a hawk on a tree at an angle of elevation of 72° If the foot of the hunter and that of the tree are on the same horizontal level and the distance between the hunter and the foot of the tree is 11 m.

(a) Represent the information on a diagram.

(b) Find the height of the tree. [Take tan 72 = 3]

Solution

(a)



(b) $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$

$\tan(72) = \frac{|BC|}{11}$

$3 = \frac{|BC|}{11}$ (multiply both,sides by 11)

$11 \times 3 = 11 \times \frac{|BC|}{11}$

$33 = |BC|$

The height of the tree = 33 m + 1.62 m

= 34.62 m

BEST BRAIN

EXERCISE 202 Date

1. The angle of elevation of a pastor from a member siting at the first floor in the church building is 30°. Given that the distance between the pastor and the member is 25 m and the height of the pastor can be ignored, find the vertical height between the foot of the pastor and the floor of the building. [Take sin(30)° = $\frac{1}{2}$]

2. A farmer saw a deer on top of a hill. The foot of the farmer is on the same horizontal level as the base of the hill. If the angle of elevation of the farmer from the deer is 50° and the distance between the farmer and the base of the hill is 20m,
[Take tan(50) = 1.2°]
(a) represent the information on a diagram;
(b) find the vertical height of the hill, ignore the height of the farmer.

3. A boy observes a friend on a storey building at an angle of elevation of 30°. If the distance between the boy and the foot of the building is 15m, calculate the vertical height of the building. [Take sin(30) = 0.8°]

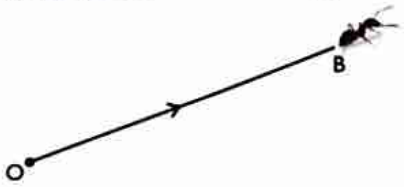
4. A hunter of height 1.7m observed a bird on top of a tree at an angle of elevation of 37° The distance between the foot of the hunter and the foot of the tree is 18m.
(a) Represent the information on a diagram.
(b) Find the distance between the hunter and the bird.
(c) Find the height of the tree
[Take sin(37) = 0.76°]

BEST BRAIN

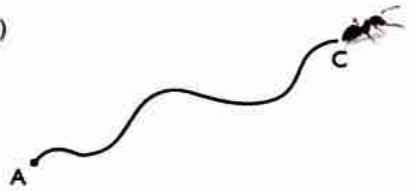
B8 3.2.1.6 Perform addition and subtraction of vectors

Look at the movements of the ants in (a) and (b).

(a)



(b)



The ant in (a) moved from point O and travelled straight to point B.

The ant in (b) moved from point A and wound through to point C.

We can measure the displacement (distance) from O to B and also find the direction in which the ant travelled. In (b) we can find the distance travelled by the ant but we cannot find its direction because the ant was winding. We therefore say that the movement of the ant in (a) is a vector quantity and the movement of the ant in (b) is a scalar quantity.

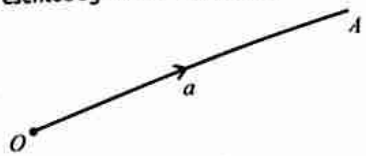
Some other quantities that describe a vector quantity are: displacement, velocity, acceleration, etc.

Some other quantities that describe a scalar quantity are: distance, speed, mass, time, etc.

A vector is usually written as $\begin{pmatrix} a \\ b \end{pmatrix}$ (column form), where a represents the number on the x-axis and b represents the number on the y-axis.

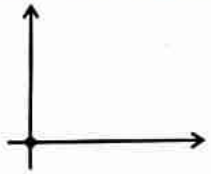
The movement of the ant in (a) is written as $\overrightarrow{OB}$. The arrow shows that the movement of the ant is from O to B. Similarly, a vector from P to Q is written as $\overrightarrow{PQ}$.

A vector quantity that starts from the origin (O) to a point is called a position vector and can be represented by a lower case letter.



The position vector OA can be written as $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$, $\overrightarrow{OC} = c$, etc.

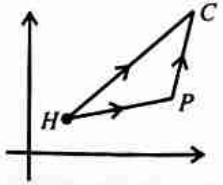
Zero vector



A zero vector is a point with no magnitude and no direction. It has a component $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Addition of vectors

Look at the displacement of Akua from her house (H) to the church (C).



BEST BRAIN

If Akua moved from H to P with vector $\overrightarrow{HP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and then from P to C with vector $\overrightarrow{PC} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$, then we can find the vector $\overrightarrow{HC}$ as an addition of $\overrightarrow{HP}$ and $\overrightarrow{PC}$ as shown in the diagram.

$$\overrightarrow{HC} = \overrightarrow{HP} + \overrightarrow{PC}$$
$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$
$$= \begin{pmatrix} 2+3 \\ 3+7 \end{pmatrix}$$
$$= \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

Example 713

Given that $\overrightarrow{MN} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\overrightarrow{NR} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$, find $\overrightarrow{MN} + \overrightarrow{NR}$

Solution

$$\overrightarrow{MN} + \overrightarrow{NR} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$
$$= \begin{pmatrix} -1 + (-3) \\ 2 + (-4) \end{pmatrix}$$
$$\overrightarrow{MN} + \overrightarrow{NR} = \begin{pmatrix} -1 - 3 \\ 2 - 4 \end{pmatrix}$$
$$\overrightarrow{MN} + \overrightarrow{NR} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

Example 714

Three vectors a, b and c are defined by $a = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $c = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ find $a + b + c$

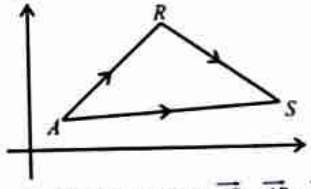
Solution

$$a + b + c = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 1 - 2 \\ 3 + 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} -1 + 4 \\ 4 + 2 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

Subtraction of vectors

Look at the movement of Fiifi from his house to the school. Fiifi moved from his house A to a point R with vector $\overrightarrow{AR} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$. He moved from the point R to the school S with vector $\overrightarrow{RS}$.

The vector from his house A to the school $\overrightarrow{AS} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$. Find the vector $\overrightarrow{RS}$.



From the diagram, we have $\overrightarrow{AS} = \overrightarrow{AR} + \overrightarrow{RS}$

$$\begin{pmatrix} 7 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \overrightarrow{RS}$$
$$\begin{pmatrix} 7 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \overrightarrow{RS}$$
$$\begin{pmatrix} 7 - 2 \\ 8 - (-2) \end{pmatrix} = \overrightarrow{RS}$$
$$\begin{pmatrix} 5 \\ 10 \end{pmatrix} = \overrightarrow{RS}$$

BEST BRAIN

Example 715

If $\overrightarrow{PQ} = \begin{pmatrix}3\\2\end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix}6\\4\end{pmatrix}$, find $\overrightarrow{PR} - \overrightarrow{PQ}$

Solution

$$\overrightarrow{PR} - \overrightarrow{PQ} = \begin{pmatrix}6\\4\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix}$$
$$= \begin{pmatrix}6-3\\4-2\end{pmatrix}$$
$$= \begin{pmatrix}3\\2\end{pmatrix}$$

Example 716

Three vectors p, q and r have components $\mathbf{p} = \begin{pmatrix}-3\\-4\end{pmatrix}$, $\mathbf{q} = \begin{pmatrix}2\\-2\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}3\\5\end{pmatrix}$. Find $\mathbf{p} - \mathbf{q} + \mathbf{r}$

Solution

$$\mathbf{p} - \mathbf{q} + \mathbf{r} = \begin{pmatrix}-3\\-4\end{pmatrix} - \begin{pmatrix}2\\-2\end{pmatrix} + \begin{pmatrix}3\\5\end{pmatrix}$$
$$= \begin{pmatrix}-3-2\\-4-(-2)\end{pmatrix} + \begin{pmatrix}3\\5\end{pmatrix}$$
$$= \begin{pmatrix}-5\\-2\end{pmatrix} + \begin{pmatrix}3\\5\end{pmatrix}$$
$$= \begin{pmatrix}-5+3\\-2+5\end{pmatrix}$$
$$= \begin{pmatrix}-2\\3\end{pmatrix}$$

EXERCISE 203 Date

1. Given that $\overrightarrow{AB} = \begin{pmatrix}1\\7\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}1\\-3\end{pmatrix}$, find $\overrightarrow{AB} + \overrightarrow{BC}$

2. If $\overrightarrow{MN} = \begin{pmatrix}-2\\3\end{pmatrix}$ and $\overrightarrow{NR} = \begin{pmatrix}3\\3\end{pmatrix}$, find $\overrightarrow{NR} + \overrightarrow{MN}$

3. Two vectors a and b are defined by $\mathbf{a} = \begin{pmatrix}5\\2\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}-1\\-1\end{pmatrix}$. find a + b.

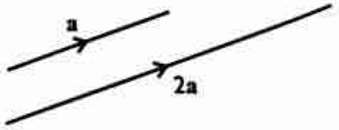
4. If $\overrightarrow{BC} = \begin{pmatrix}6\\-3\end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix}7\\-4\end{pmatrix}$, find $\overrightarrow{CD} - \overrightarrow{BC}$

5. $\overrightarrow{ST} = \begin{pmatrix}1\\4\end{pmatrix}$, $\overrightarrow{TR} = \begin{pmatrix}3\\2\end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix}-1\\-5\end{pmatrix}$, find
   (i) $\overrightarrow{TR} - \overrightarrow{RS}$
   (ii) $\overrightarrow{ST} - \overrightarrow{RS}$
   (iii) $\overrightarrow{TR} - \overrightarrow{ST} - \overrightarrow{RS}$

6. m, n and r are $\mathbf{m} = \begin{pmatrix}5\\5\end{pmatrix}$, $\mathbf{n} = \begin{pmatrix}3\\-1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}1\\1\end{pmatrix}$, find
   (i) m − n
   (ii) n − r
   (iii) m + n − r
   (iv) m − n − r

Scalar multiplication of vectors

Consider the vectors a and 2a



We can observe that the length of 2a is twice the length of a. If a has components $\begin{pmatrix}x\\y\end{pmatrix}$ then 2a has components $2\begin{pmatrix}x\\y\end{pmatrix}$ or $\begin{pmatrix}2x\\2y\end{pmatrix}$.

The number 2 is called a scalar.

BEST BRAIN

Example 717

Two vectors are defined by $\mathbf{p} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix}1\\2\end{pmatrix}$, find p + 2q.

Solution

$$\mathbf{p} + 2\mathbf{q} = \begin{pmatrix}2\\3\end{pmatrix} + 2\begin{pmatrix}1\\2\end{pmatrix}$$
$$= \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}2\times1\\2\times2\end{pmatrix}$$
$$= \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}2\\4\end{pmatrix}$$
$$= \begin{pmatrix}2+2\\3+4\end{pmatrix}$$
$$= \begin{pmatrix}4\\7\end{pmatrix}$$

Example 718

$\mathbf{m} = \begin{pmatrix}2\\-1\end{pmatrix}$, $\mathbf{n} = \begin{pmatrix}1\\2\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}-2\\5\end{pmatrix}$, find 2m + 3n − r.

Solution

$$2\mathbf{m} + 3\mathbf{n} - \mathbf{r} = 2\begin{pmatrix}2\\-1\end{pmatrix} + 3\begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}-2\\5\end{pmatrix}$$
$$= \begin{pmatrix}4\\-2\end{pmatrix} + \begin{pmatrix}3\\6\end{pmatrix} - \begin{pmatrix}-2\\5\end{pmatrix}$$
$$= \begin{pmatrix}7\\4\end{pmatrix} - \begin{pmatrix}-2\\5\end{pmatrix}$$
$$= \begin{pmatrix}9\\-1\end{pmatrix}$$

Example 719

$\mathbf{x} = \begin{pmatrix}3\\-5\end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix}-5\\3\end{pmatrix}$, find $\frac{1}{2}(\mathbf{x} - 3\mathbf{y})$

Solution

$$(\mathbf{x} - 3\mathbf{y}) = \begin{pmatrix}3\\-5\end{pmatrix} - 3\begin{pmatrix}-5\\3\end{pmatrix}$$
$$= \begin{pmatrix}3\\-5\end{pmatrix} - \begin{pmatrix}-15\\9\end{pmatrix}$$ (use 3 to multiply −5 and 3)
$$= \begin{pmatrix}3-(-15)\\-5-9\end{pmatrix}$$
$$= \begin{pmatrix}3+15\\-14\end{pmatrix}$$
$$= \begin{pmatrix}18\\-14\end{pmatrix}$$
$$\therefore \frac{1}{2}(\mathbf{x} - 3\mathbf{y}) = \frac{1}{2}\begin{pmatrix}18\\-14\end{pmatrix}$$
$$= \begin{pmatrix}\frac{1}{2}\times18\\\frac{1}{2}\times-14\end{pmatrix}$$
$$= \begin{pmatrix}9\\-7\end{pmatrix}$$

EXERCISE 204 Date

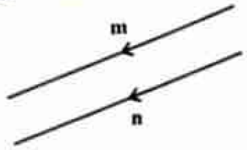
$\mathbf{s} = \begin{pmatrix}4\\7\end{pmatrix}$, $\mathbf{r} = \begin{pmatrix}2\\5\end{pmatrix}$, $\mathbf{t} = \begin{pmatrix}1\\-1\end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix}-2\\-3\end{pmatrix}$, find

(a) s + 2r
(b) 2r − u
(c) 3r − 2s
(d) $\frac{1}{2}$s + 3t
(e) 3s − 2t + u
(f) 2r − 3t
(g) $\frac{1}{2}$s + u
(h) s + r − u
(i) 5t − s − u
(j) $\frac{1}{2}$(s + t) − r

BEST BRAIN

B8.3.2.1.7 Demonstrate understanding of equal vectors

Consider the vectors



We can observe that the vectors m and n have equal length and in the same direction. We can therefore say that the vectors are equal, i.e.

$\mathbf{m} = \mathbf{n}$.

When two vectors are equal, then their corresponding components are also equal.

Example 720

The vectors $\mathbf{a} = \begin{pmatrix} 2 \\ y+2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ are equal, find the value of y.

Solution

$\mathbf{a} = \mathbf{b}$

$\begin{pmatrix} 2 \\ y+2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$y + 2 = 3$

$y = 3 - 2$

$y = 1$

Example 721

Given that $\mathbf{m} = \begin{pmatrix} x-3 \\ x-y \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} -2 \\ 2x+1 \end{pmatrix}$, find the value of x and y if $\mathbf{m} = \mathbf{n}$.

Solution

$\begin{pmatrix} x-3 \\ x-y \end{pmatrix} = \begin{pmatrix} -2 \\ 2x+1 \end{pmatrix}$

$x - 3 = -2$ .....(1)

$x - y = 2x + 1$ .....(2)

From (1)

$x - 3 = -2$

$x = -2 + 3$

$x = 1$

From (2)

$x - y = 2x + 1$

$1 - y = 2(1) + 1$

$1 - y = 2 + 1$

$1 - y = 3$

$1 = 3 + y$

$1 - 3 = y$

$-2 = y$

Example 722

The vectors p, q and r are $\mathbf{p} = \begin{pmatrix} x \\ 5 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} x+y \\ y \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$. Given that $\mathbf{q} = \mathbf{r} - \mathbf{p}$, find the values of x and y.

Solution

$\mathbf{r} - \mathbf{p} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} x \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 5-x \\ 2-5 \end{pmatrix}$

$\mathbf{q} = \mathbf{r} - \mathbf{p}$

$\begin{pmatrix} x+y \\ y \end{pmatrix} = \begin{pmatrix} 5-x \\ 2-5 \end{pmatrix}$

$x + y = 5 - x$ .....(1)

$y = 2 - 5$ .....(2)

From (2)

$y = 2 - 5$

$y = -3$

From (1)

$x + y = 5 - x$

$x + (-3) = 5 - x$

$x - 3 = 5 - x$

$x + x - 3 = 5 - x + x$

$2x - 3 = 5$

$2x = 5 + 3$

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

EXERCISE 205 Date

1. $\mathbf{m} = \begin{pmatrix} -2 \\ 2y-3 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} -1+x \\ 1 \end{pmatrix}$, and $\mathbf{m} = \mathbf{n}$, find the values of x and y.

2. Given that $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3x-2 \\ 2 \end{pmatrix}$, find the value of x if $\mathbf{a} = \mathbf{b}$.

3. If $\mathbf{t} = \begin{pmatrix} 5 \\ y-3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 5 \\ 1-y \end{pmatrix}$, find the value of y, given that $\mathbf{t} = \mathbf{r}$.

4. If $\mathbf{m} = \begin{pmatrix} 9 \\ -12 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} 5-x \\ y+3 \end{pmatrix}$, find the values of x and y if $\frac{1}{3}\mathbf{m} = \mathbf{n}$.

5. Given that $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} x \\ y+1 \end{pmatrix}$, find the values of x and y if $\mathbf{c} = 3\mathbf{a} - 2\mathbf{b}$.

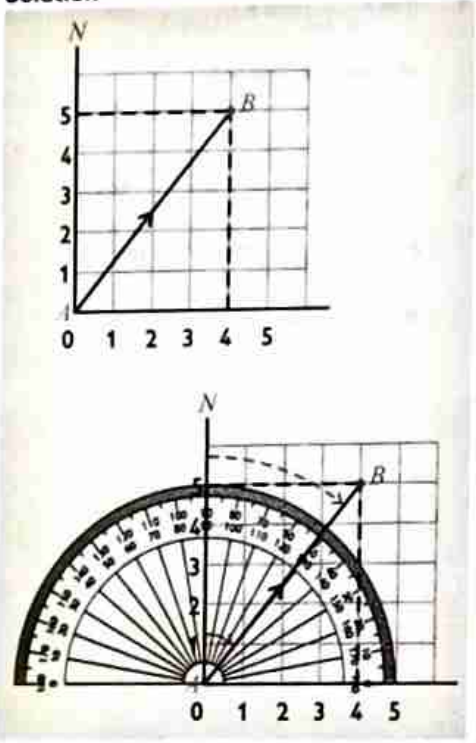
B8.3.2.1.8 Demonstrate understanding of bearings

In Basic 7, we learnt how to measure angle formed between a north pole and any adjoining line. We also learnt how to write the angle as a 3-digit number (bearing). In this section, we shall use our knowledge on vectors to express components of a vector graphically and measure its angle.

Example 723

Express the vector $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ graphically and find the length of $\overrightarrow{AB}$ and the bearing of B from A.

Solution

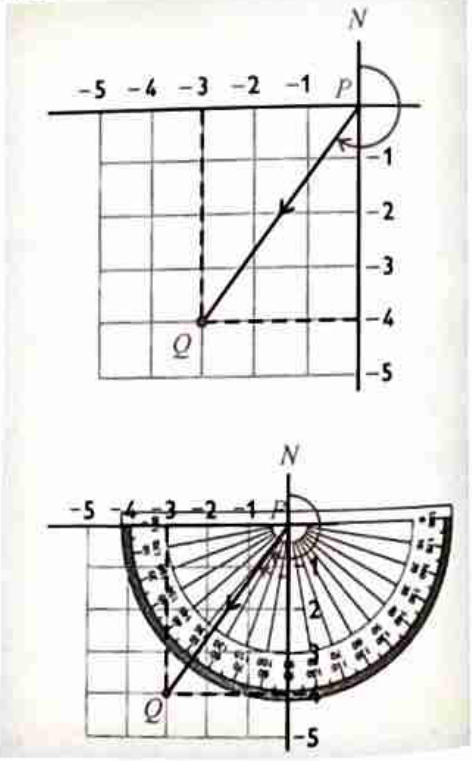


Drawing the vector on a graph sheet and using rule to measure, the length of $\overrightarrow{AB} = 6.4$ units.
Using protractor and measuring from the north, the bearing of B from A (measuring from A) is 039°.

Example 724

Express the vector $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ graphically and find the length of $\overrightarrow{PQ}$ and the bearing of Q from P.

Solution



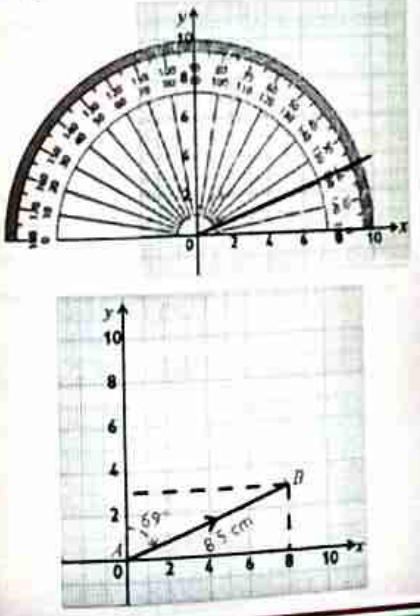
Drawing the vector on a graph sheet and using rule to measure, the length of $\overrightarrow{PQ} = 5$ units.
Using protractor and measuring from the north, the bearing of Q from P is 217°
Note: This can also be written as $\overrightarrow{PQ} = (5 \text{ units}, 217°)$

Example 725

Use 1 cm = 1 km, represent the vector bearing form $\overrightarrow{AB} = (8.5 \text{ km}, 069°)$ graphically and write $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$.

Solution

(i) First draw a plane and mark the origin as A.
(ii) Measure an angle of 69°, using protractor, starting from the north (N).
(iii) Draw a line through the 69° mark and measure 8.5 cm from A, mark the end as B.



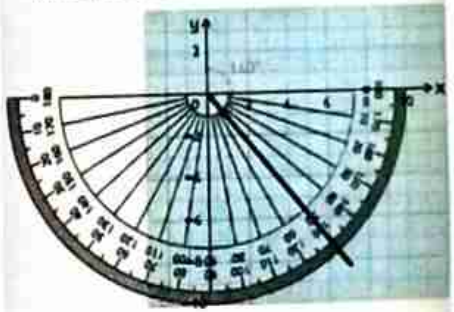
Therefore $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

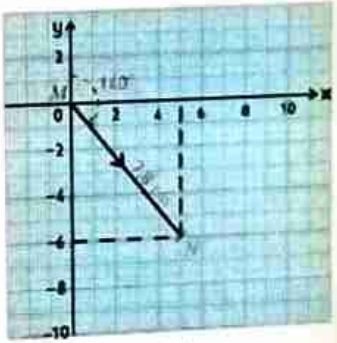
Example 726

Use 1 cm = 1 km, represent the vector bearing form $\overrightarrow{MN} = (7.8 \text{ km}, 140°)$ graphically and write $\overrightarrow{MN} = \begin{pmatrix} x \\ y \end{pmatrix}$.

Solution

(i) First draw a plane and mark the origin as M.
(ii) Measure an angle of 140°, using protractor, starting from the north (N)
(iii) Draw a line through the 140° mark and measure 7.8 cm from M, mark the end as N.



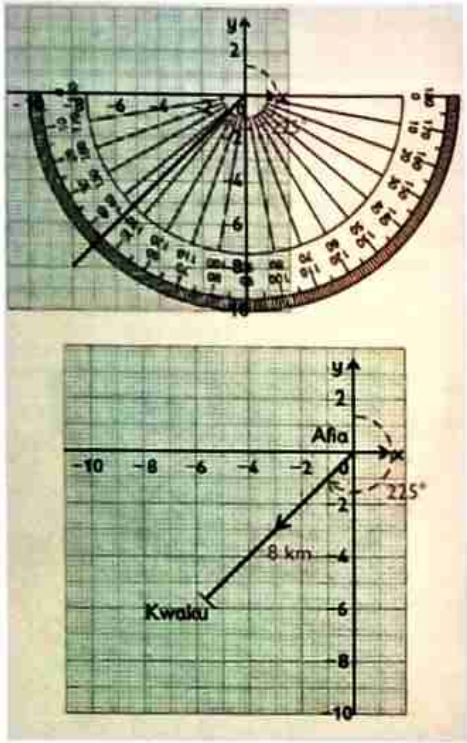


Therefore $\overrightarrow{MN} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$

Example 727

Afia and Kwaku are at two different points in a plane 8 km apart. The bearing of Kwaku from Afia is 225°. Represent their positions on a diagram. [Take 1 cm = 1 km]

Solution



EXERCISE 206 Date.....

1. Express the vector $\overrightarrow{EF} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ graphically and find the length of $\overrightarrow{EF}$ and the bearing of F from E.

2. Express the vector $\overrightarrow{MN} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$ graphically and find the length of $\overrightarrow{MN}$ and the bearing of N from M.

3. Use 1 cm = 1 km, represent the vector bearing form $\overrightarrow{PQ} = (9.5\text{ km}, 238°)$ graphically and write $\overrightarrow{PQ} = \begin{pmatrix} x \\ y \end{pmatrix}$.

4. Use 1 cm = 1 km, represent the vector bearing form $\overrightarrow{AB} = (8.9\text{ km}, 153°)$ graphically and write $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$.

5. Atinga and Naa are at two different points in a plane 10 km apart. The bearing of Naa from Atinga is 110°. Represent their positions on a diagram. [Take 1 cm = 1 km]

B8.3.2.1.9 Demonstrate understanding of back bearing

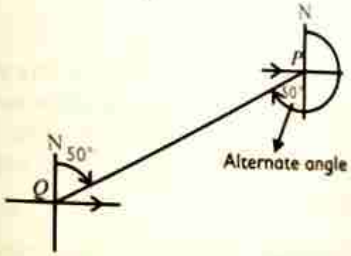
In the previous section, we learnt how to find the bearing of a point say A from another point say B. In this section, we shall learn how to find the bearing of each other point, i.e. the bearing of A from B and the bearing of B from A. If the bearing of A from B is known, then the bearing of B from A called back bearing can also be found.

Example 728

The bearing of P from Q is 050°, find the bearing of Q from P.

Solution

(a) Draw a cardinal plane.
(b) Measure 50° from the north and mark it Q (not necessarily using a protractor). Measure from the North Pole to the right.
(c) Draw any reasonable line through the 50° point and mark it P.
(d) Use alternate angle to measure the bearing from Q.



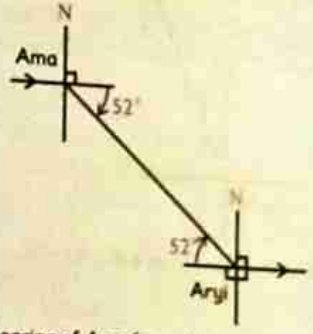
The bearing of Q from $P = 180° + 50°$
$= 230°$

Example 729

The bearing of Aryi from Ama is 142°, find the bearing of Ama from Aryi.

Solution

Using the same method as outlined in example 1
$142° = 90° + 52°$



The bearing of Ama from Aryi
$= 90° + 90° + 90° + 52°$
$= 180° + 142°$
$= 322°$

From the two preceding examples, we can conclude that when the bearing of a point from another point say $x°$, where $x°$ is less than 180°, then the back bearing is $180° + x°$.

Example 730

The bearing of C from D is 084°, find the bearing of D from C.

Solution

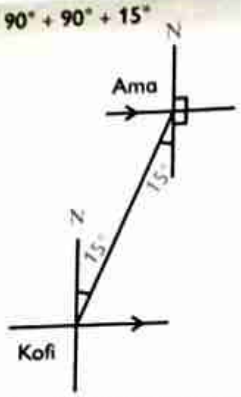
Since $084° < 180°$ then,
Bearing of D from $C = 180° + 84°$
$= 264°$

Example 731

The bearing of Kofi from Ama is 195°, find the bearing of Ama from Kofi.

Solution

$195° = 90° + 90° + 15°$



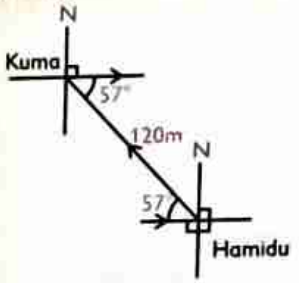
The bearing of Ama from Kofi = 015° or
= 195° − 180°
= 015°

Example 732

The distance between Hamidu and Kuma is 120 m, the bearing of Kuma from Hamidu is 327°, find the bearing of Hamidu from Kuma.

Solution

$327° = 90° + 90° + 90° + 57°$



The bearing of Hamidu from Kuma
= 90° + 57°
= 147° or
= 327° − 180°
= 147°

From the preceding two examples, we can conclude that when the bearing of a point from another point say $x°$, where $x°$ is greater than 180°, then the back bearing is $x° - 180°$.

Example 733

The bearing of E from F is 280°, find the bearing of F from E.

Solution

Bearing of F from $E = 280° - 180° = 100°$

EXERCISE 207 Date

1. Two points A and B are in a plane such that the bearing of A from B is 093°, find the bearing of B from A.
2. The bearing of house C from house H is 153°, what is the bearing of house H from house C?
3. Two trees X and Y are in the same plane such that the bearing of X from Y is 037°, find the bearing of Y from X.
4. The distance between a school and a police station is 2 km. The bearing of the police station from the school is 220°. Find the bearing of the school from the police station.
5. A church house and a mosque are in the same plane with a distance of 500m. The bearing of the church from the mosque is 311°, find the bearing of the mosque from the church.
6. The bearing of Amina from Akuba is 200°, find the bearing of Akuba from Amina.

BEST BRAIN

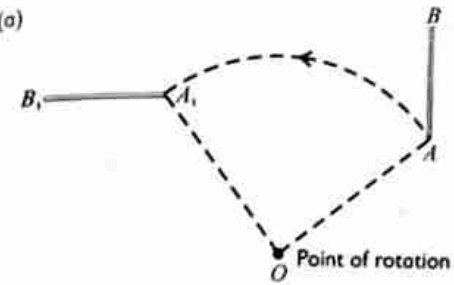
STRAND 3.3

POSITION AND TRANSFORMATION

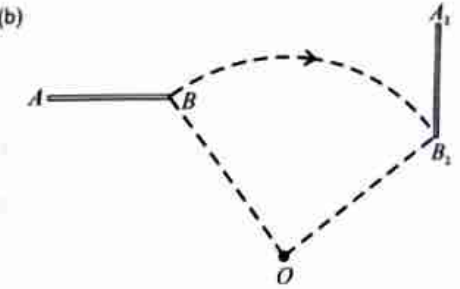
B8.3.3.1.1 Demonstrate understanding of rotation

Look at the position of AB, A_1B_1 and A_2B_2 from the point O.

(a)



(b)

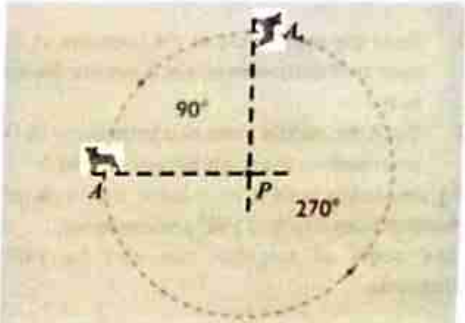


In (a), the object AB has been rotated to its new position A_1B_1 through the point of rotation O. The rotation of AB to A_1B_1 is anticlockwise.
Similarly, in (b), the object BA has been rotated to its new position B_2A_2 through the point of rotation O. The rotation of BA to B_2A_2 is clockwise
In this section, we shall use the idea of rotation to describe position of objects.

Example 734

The cub at position A moved to a new position A_1. Describe the position of A_1 with respect to A given that P is the point of rotation.

Solution



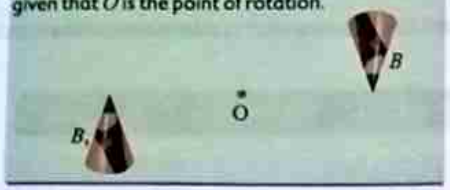
1. Place the metal part of the compass at P, open to A and swing an arc in anticlockwise to A_1.
2. Place the middle point of a protractor at P and measure the angle between A and A_1.

By measuring with a protractor, the angle of rotation from A to A_1 is 270° anticlockwise.
The angle of rotation can also be 90° clockwise.

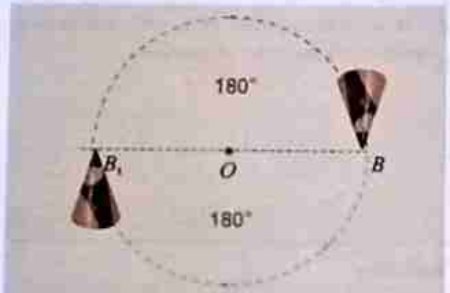
BEST BRAIN

Example 735

Describe the position of the cone at B and B_1 given that O is the point of rotation.



Solution



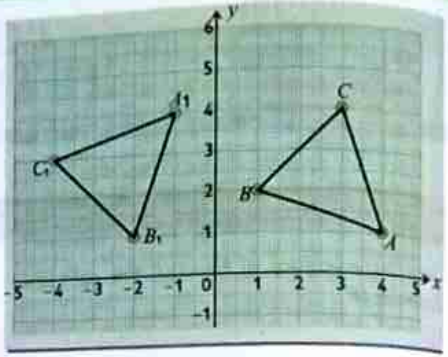
1. Place the metal part of the compass at O, open to B and swing an arc in anticlockwise to B_1.
2. Place the middle point of a protractor at O and measure the angle between B and B_1.

By measuring with a protractor, the angle of rotation from B to B_1 is 180° anticlockwise.

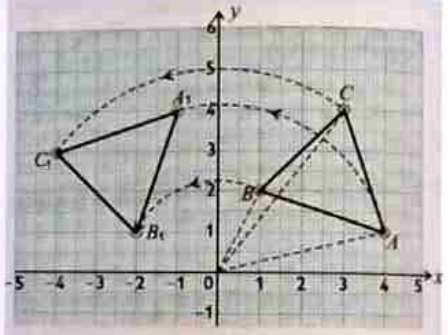
The angle of rotation can also be 180° clockwise.

Example 736

Describe the position of the triangle ABC and $A_1B_1C_1$ given that 0 is the point of rotation.



Solution



1. Place the metal part of the compass at 0, open to B and swing an arc in anticlockwise to B_1.
2. Place the middle point of a protractor at 0 and measure the angle between B and B_1.

By measuring with a protractor, the angle of rotation from B to B_1 is 90° anticlockwise.

The angle of rotation can also be 270° clockwise.

Similar measurement can be done from A to A_1 and C to C_1 to produce the same angle.

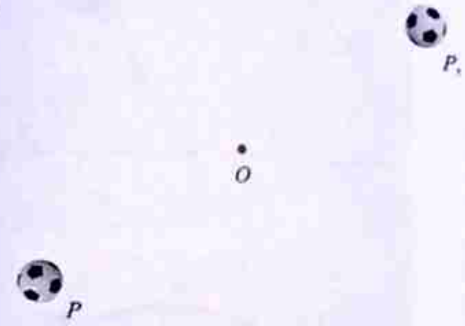
Therefore triangle ABC is rotated through 90° anticlockwise to produce triangle $A_1B_1C_1$.

BEST BRAIN

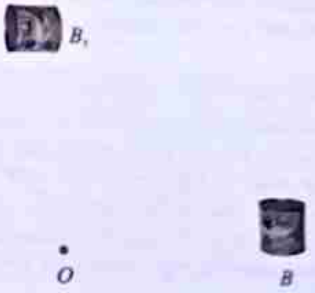
EXERCISE 208

1. (a) Describe the position of P_1 with respect to P given that O is the point of rotation.
   (b) Describe the position of B_1 with respect to B given that O is the point of rotation.

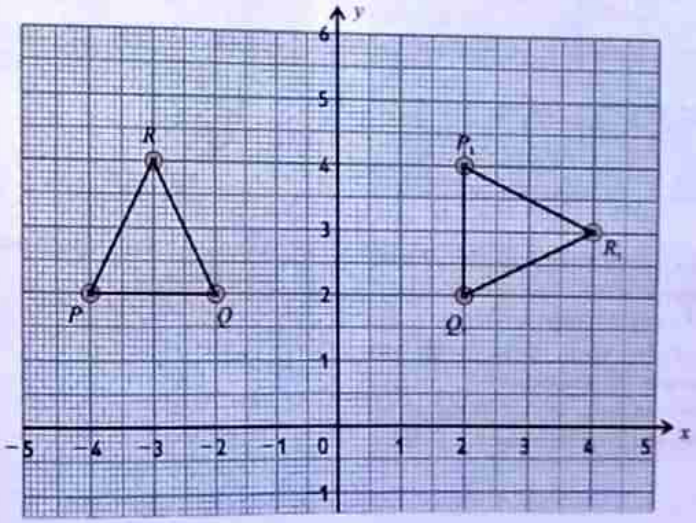
(a)



(b)



2. The triangle PQR is rotated to produce triangle $P_1Q_1R_1$, describe the two forms of rotation that produced triangle $P_1Q_1R_1$.



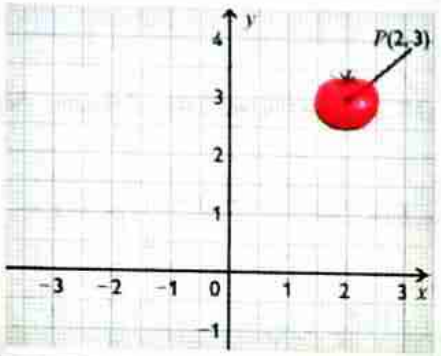
BEST BRAIN

B8.3.3.1.2 Draw image in a coordinate plane using rotation

In this section, we shall use our knowledge from the previous section on rotation to draw image in a coordinate plane, determine the angle of rotation and deduce the rule for rotation of 90°, 180° and 270° about the origin.

Example 737

A tomato has its center $P(2, 3)$ in a coordinate plane as shown. Use a protractor to measure the angle of rotation of 90°, 180° and 270° anticlockwise. Write the center of the new position P_1 and deduce the relation between P and P_1 with the origin as the point of rotation.

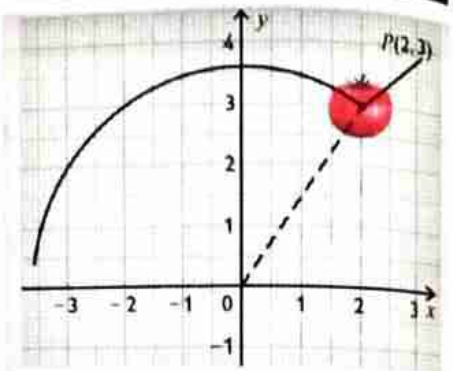


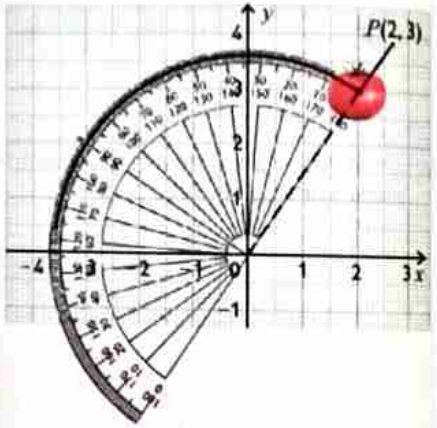
Solution

(a) Rotation of 90° anticlockwise or 270° clockwise.

(i) With 0 as the center, open the compass to P the center of the tomato and swing an arc anticlockwise.

(ii) Place the center of a protractor at 0 in line with OP and measure 90°.



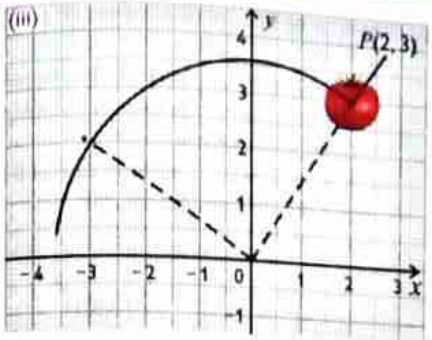


(iii) Join a line from 0 through the 90° mark to meet the arc in (i) at P_1.

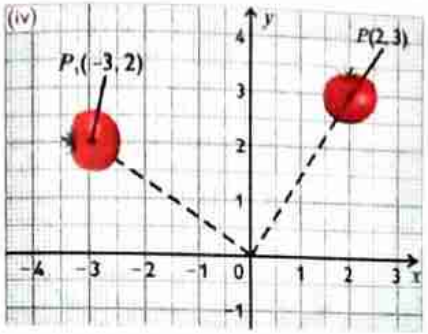
(iv) Write the coordinates of P_1.

BEST BRAIN

(iii)



(iv)



From (iv) the coordinates of P_1 is $(-3, 2)$.

From P to P_1

$P(2, 3) \rightarrow (x, y)$

$P_1(-3, 2) \rightarrow (-y, x)$

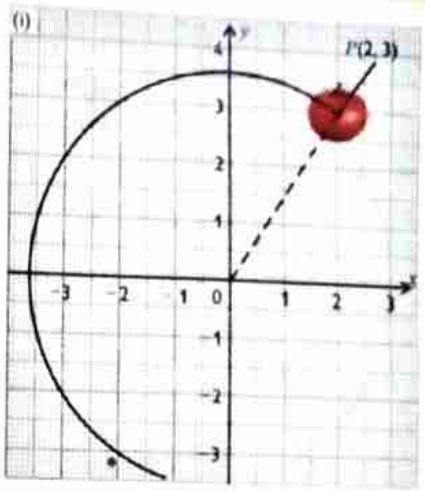
$\therefore$ Rotation of 90° anticlockwise or 270° clockwise $(x, y) \rightarrow (-y, x)$.

(b) Rotation of 180° anticlockwise or −180° clockwise.

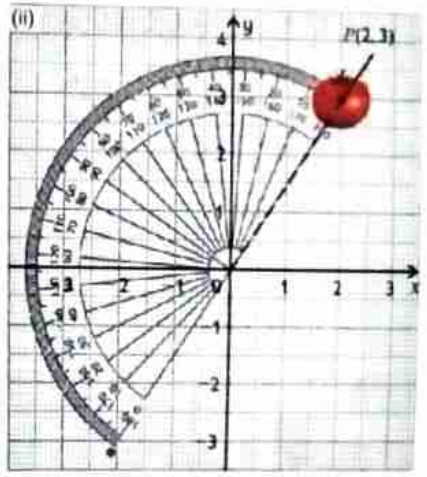
(i) With 0 as the center, open the compass to P the center of the tomato and swing an arc anticlockwise.

(ii) Place the center of a protractor at 0 in line with OP and measure 180°.

(i)



(ii)

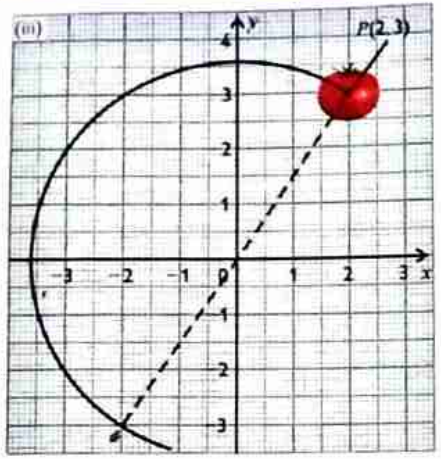


(iii) Join a line from 0 through the 180° mark to meet the arc in (i) at P.

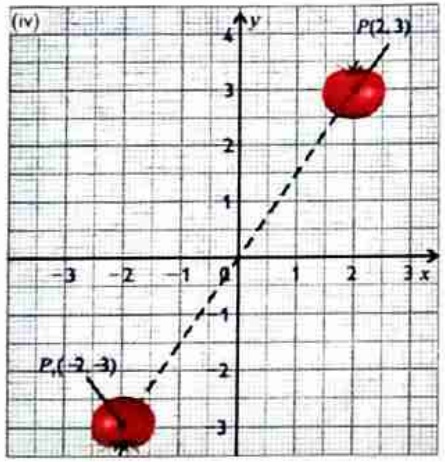
(iv) Write the coordinates of P.

BEST BRAIN

(iii)



(iv)



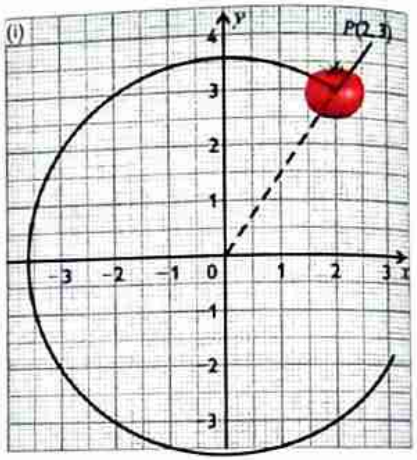
From (iv) the coordinates of P_1 is $(-2, -3)$.
From P to P_1
$P(2,3) \rightarrow (x,y)$
$P(-2,-3) \rightarrow (-x,-y)$
∴ Rotation of 180° anticlockwise or clockwise $(x,y) \rightarrow (-x,-y)$.

(c) Rotation of 270° anticlockwise or −270° clockwise.

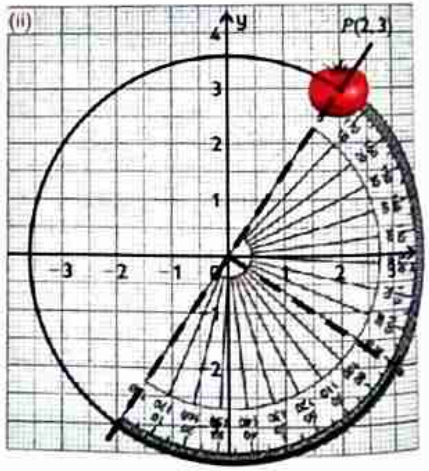
(i) With 0 as the center, open the compass to P the center of the tomato and swing an arc anticlockwise.

(ii) Place the center of a protractor at 0 in line with OP and measure 270°.

(i)



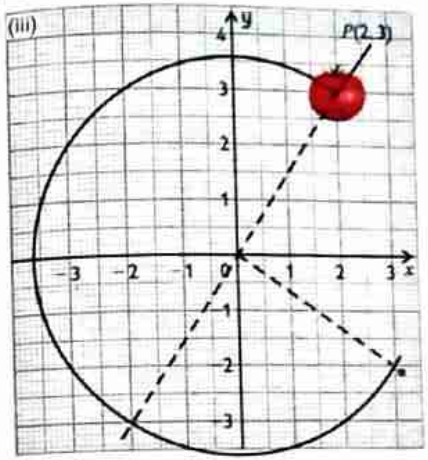
(ii)



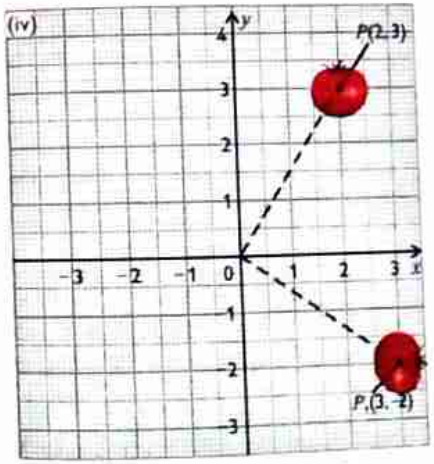
(iii) Join a line from 0 through the 270° mark to meet the arc in (i) at P_1.

(iv) Write the coordinates of P_1.

(iii)



(iv)



From (iv) the coordinates of P_1 is $(3, -2)$.
From P to P_1
$P(2,3) \rightarrow (x,y)$
$P_1(3,-2) \rightarrow (y,-x)$
∴ Rotation of 270° anticlockwise or 90° clockwise $(x,y) \rightarrow (y,-x)$.

Example 738

(i) Using a scale of 2 cm to 1 unit on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) On this graph sheet, mark the x-axis from −5 to 5 and on the y-axis from −6 to 6.
Plot on the same graph sheet the points $A(2,1)$, $B(1,3)$ and $C(3,3)$. Join the points to form a triangle ABC.

(iv) Draw the image triangle $A_1B_1C_1$ of ABC under an anticlockwise rotation of 180° about the origin, where $A \rightarrow A_1$, $B \rightarrow B_1$ and $C \rightarrow C_1$.

(v) Draw the image triangle $A_2B_2C_2$ of ABC under an anticlockwise rotation of 270° about the origin, where $A \rightarrow A_2$, $B \rightarrow B_2$ and $C \rightarrow C_2$.

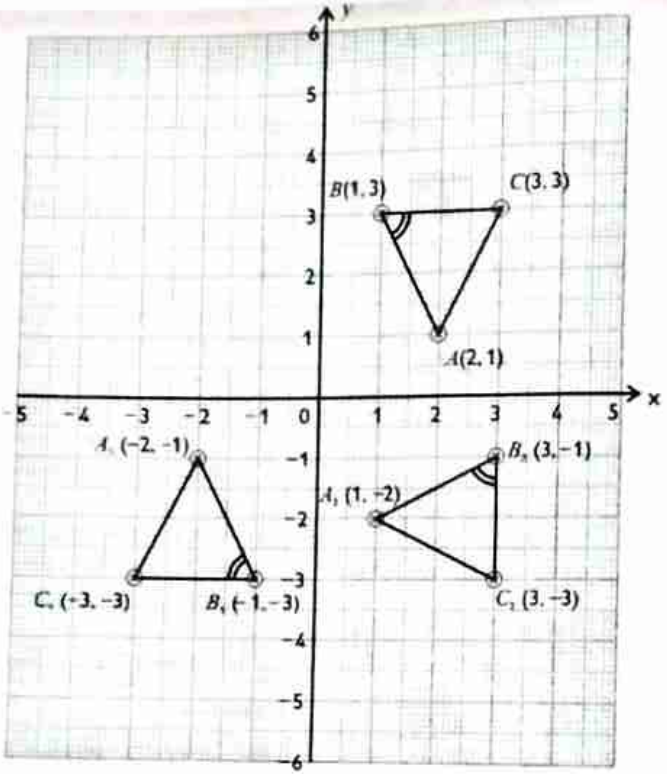
(vi) Compare angle ABC, $A_1B_1C_1$ and $A_2B_2C_2$.

Solution

(iii) Draw the perpendicular axes, label the axes and plot the points ABC with their coordinates written on them.

(iv) Rotation through 180° anticlockwise.
$(x,y) \rightarrow (-x,-y)$
$A(2,1) \rightarrow A_1(-2,-1)$, $B(1,3) \rightarrow B_1(-1,-3)$
$C(3,3) \rightarrow C_1(-3,-3)$
Plot the points $A_1B_1C_1$ on the graph and label.

(v) Rotation through 270° anticlockwise.
$(x,y) \rightarrow (y,-x)$
$A(2,1) \rightarrow A_2(1,-2)$ $B(1,3) \rightarrow B_2(3,-1)$
$C(3,3) \rightarrow C_2(3,-3)$
Plot the points $A_2B_2C_2$ on the graph and label.



(vi) From the graph angle ABC, $A_1B_1C_1$ and $A_2B_2C_2$ are equal. It means that even though their positions are different, they are all congruent.

Example 739

(a) (i) Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) On this graph sheet, mark the x-axis from −10 to 10 and on the y-axis from −12 to 12.

(iii) Plot on the same graph sheet the points $M(2, -4)$, $N(7, -4)$ and $R(7, -9)$. Join the points to form a triangle MNR.

(iv) Draw the image triangle $M_1N_1R_1$ under an clockwise rotation of 90° about the origin, where $M \to M_1$, $N \to N_1$ and $R \to R_1$.

(v) Draw the image $M_2N_2R_2$ of triangle MNR under a reflection in the x-axis where $M \to M_2$, $N \to N_2$ and $R \to R_2$.

(b) Find the gradient of M_1 and N_2

BEST BRAIN

Solution

(a) (i) Draw the perpendicular axes and label the axes.

(iii) Plot the points MNR with their coordinates written on them.

(iv) Rotation through 90° clockwise,

$(x, y) \to (y, -x)$

$M(2, -4) \to M_1(-4, -2)$

$N(7, -4) \to N_1(-4, -7)$

$R(7, -9) \to R_1(-9, -7)$

Plot the points $M_1N_1R_1$ on the graph and label.

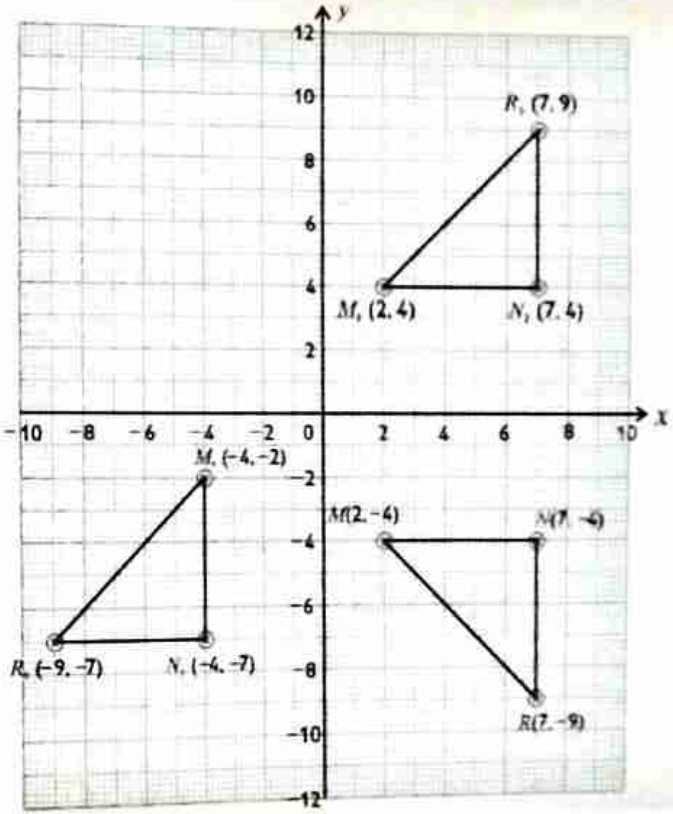
(v) Use the knowledge on reflection to plot the points $M_2N_2R_2$.

(b) $M_2 = (-4, -2)$ and $N_2 = (7, 4)$

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{7 - (-4)}$$

$$= \frac{4 + 2}{7 + 4}$$

$$= \frac{6}{11}$$



BEST BRAIN

Example 740

(i) Using a scale of 2 cm to 5 units on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) On this graph sheet, mark the x-axis from -25 to 25 and on the y-axis from -30 to 30.

(iii) Plot on the same graph sheet the points $P(-5, -10)$, $Q(-15, -10)$, $R(-20, -20)$ and $S(-10, -20)$. Join the points to form a quadrilateral.

(iv) Draw the image $P_1Q_1R_1S_1$ of $PQRS$ under an anticlockwise rotation of 90° where $P \to P_1, Q \to Q_1, R \to R_1$ and $S \to S_1$.

(v) Draw the image $P_2Q_2R_2S_2$ of $PQRS$ under a translation by the vector $\begin{pmatrix} -5 \\ 35 \end{pmatrix}$ where $P \to P_2, Q \to Q_2, R \to R_2$ and $S \to S_2$.

Solution

(a) (i) Draw the perpendicular axes and label the axes.

(iii) Plot the points $PQRS$ with their coordinates written on them.

(iv) Rotation through 90° anticlockwise,

$(x, y) \to (-y, x)$

$P(-5, -10) \to P_1(10, -5)$

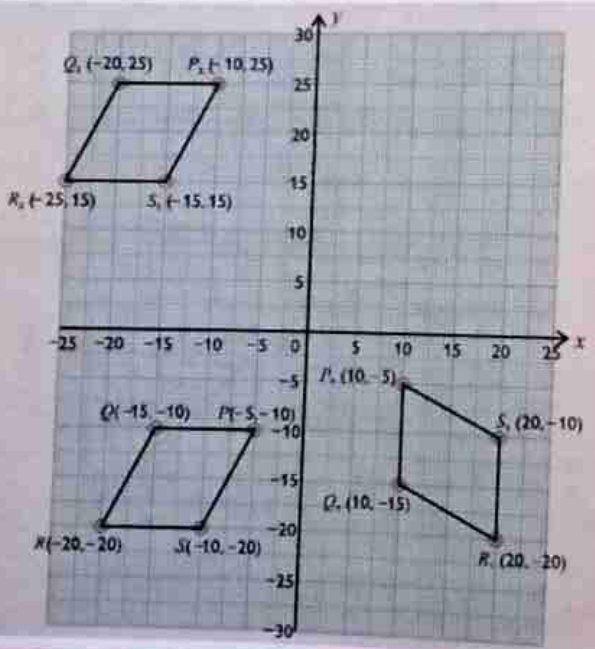
$Q(-15, -10) \to Q_1(10, -15)$

$R(-20, -20) \to R_1(20, -20)$

$S(-10, -20) \to S_1(20, -10)$

Plot the points $P_1Q_1R_1S_1$ on the graph and label.

(v) Use the knowledge on translation to plot the points $P_2Q_2R_2S_2$.



EXERCISE 209 Done

1. (a) (i) Using a scale of 2 cm to 1 unit on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) On this graph sheet, mark the x-axis from -5 to 5 and on the y-axis from -6 to 6.

(iii) Plot on the same graph sheet the points $X(-4, 5)$, $Y(-1, 4)$ and $Z(-3, 2)$. Join the points to form a triangle XYZ.

(iv) Draw the image triangle $X_1Y_1Z_1$ of XYZ under a clockwise rotation of 270° about the origin, where $X \to X_1$, $Y \to Y_1$ and $Z \to Z_1$.

(v) Draw the image triangle $X_2Y_2Z_2$ of triangle XYZ under an anticlockwise rotation of 180° about the origin, where $X \to X_2, Y \to Y_2$ and $Z \to Z_2$.

(b) Find $\angle ZXY$, $\angle Z_1X_1Y_1$ and $\angle Z_2X_2Y_2$ and comment on the answer.

2. (a) (i) Using a scale of 2 cm to 1 unit on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) On this graph sheet, mark the x-axis from -5 to 5 and on the y-axis from -6 to 6.

(iii) Plot on the same graph sheet the points $S(3, -1)$, $T(5, -4)$ and $R(1, -4)$. Join the points to form a triangle STR.

(iv) Draw the image triangle $S_1T_1R_1$ of triangle STR under a clockwise rotation of 90° about the origin, where $S \to S_1$, $T \to T_1$ and $R \to R_1$.

(v) Draw the image $S_2T_2R_2$ of triangle STR under a reflection in the line $y = 0$ where $S \to S_2, T \to T_2$ and $R \to R_2$.

(b) What type of triangle is $S_1T_1R_1$?

(c) Measure angle $S_1T_1R_1$.

3. (a) (i) Using a scale of 2 cm to 10 units on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) On this graph sheet, mark the x-axis from -50 to 50 and on the y-axis from -60 to 60.

(iii) Plot on the same graph sheet the points $C(10, 20)$, $D(35, 30)$ and $E(20, 50)$. Join the points to form a triangle.

(iv) Draw the image $C_1D_1E_1$ of CDE under an anticlockwise rotation of 90° where $C \to C_1$, $D \to D_1$ and $E \to E_1$.

(v) Draw the image $C_2D_2E_2$ of $C_1D_1E_1$ under a translation by the vector $\begin{pmatrix} 65 \\ -50 \end{pmatrix}$ where $C_1 \to C_2$, $D_1 \to D_2$ and $E_1 \to E_2$.

(b) What type of triangle is $C_2D_2E_2$?

4. (a) (i) Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) Plot on the same graph sheet the points $A(0, 2)$, $B(0, 6)$, $C(4, 6)$ and $D(4, 2)$. Join the points to form a quadrilateral.

(iii) Draw the image $A_1B_1C_1D_1$ of $ABCD$ under a translation by the vector $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ where $A \to A_1, B \to B_1, C \to C_1$ and $D \to D_1$.

(iv) Draw the image $A_2B_2C_2D_2$ of $A_1B_1C_1D_1$ under a rotation of 180° about the origin where $A_1 \to A_2$, $B_1 \to B_2, C_1 \to C_2$ and $D_1 \to D_2$.

(b) Calculate the slope of C_2D_2.

5. (i) Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) Plot on the same graph sheet the points $P(-9,-11)$, $Q(-5,-1)$ and $R(-5,-11)$. Join the points to form a triangle.

(iii) Draw the image $P_1Q_1R_1$ of PQR under a clockwise rotation of 270° about the origin where $P \rightarrow P_1$, $Q \rightarrow Q_1$ and $R \rightarrow R_1$.

(iv) Draw the image $P_2Q_2R_2$ of PQR under a translation by the vector $\begin{pmatrix} 3 \\ 13 \end{pmatrix}$ where $P \rightarrow P_2$, $Q \rightarrow Q_2$ and $R \rightarrow R_2$.

(v) Draw the image $P_3Q_3R_3$ of $P_1Q_1R_1$ under a reflection in the line $y = 0$.

BEST BRAIN

STRAND 4: HANDLING DATA

SUB-STRANDS

- Data
- Chance or probability

(b) Calculate the slope of C_2D_2.

5. (i) Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes OX and OY on a graph sheet.

(ii) Plot on the same graph sheet the points $P(-9,-11)$, $Q(-5,-1)$ and $R(-5,-11)$. Join the points to form a triangle.

(iii) Draw the image $P_1Q_1R_1$ of PQR under a clockwise rotation of 270° about the origin where $P \rightarrow P_1$, $Q \rightarrow Q_1$ and $R \rightarrow R_1$.

(iv) Draw the image $P_2Q_2R_2$ of PQR under a translation by the vector $\begin{pmatrix} 3 \\ 13 \end{pmatrix}$ where $P \rightarrow P_2$, $Q \rightarrow Q_2$ and $R \rightarrow R_2$.

(v) Draw the image $P_3Q_3R_3$ of $P_1Q_1R_1$ under a reflection in the line $y = 0$.

STRAND 4:
HANDLING DATA

SUB-STRANDS

- Data
- Chance or probability

STRAND 4.1

DATA

B8.4.1.1.1 Identify types of data

In this section, we shall identify types of data one can obtain from a survey. Basically, there are two types of data, namely: categorical and numerical.

Categorical data

Categorical data describes categories of groups. An example is car brands such as Nissan, Toyota, Mercedes Benz, etc. Another example of categorical data is answers to yes and no questions.

Categorical data is qualitative, that is, it describes an event using words rather than using numbers.

Categorical data is collected using questionnaire.

Types of Categorical data

There are two types of categorical data namely: nominal and ordinal data.

Nominal data

This is a type of data used to name variables without providing any numerical value. Example: name, hair colour, sex, etc.

Ordinal data

This is a data with a scale to it. For example: excellent, very good, good, poor, etc. We usually use a numerical scale 1, 2, 3, 4, etc.

This scale is called Likert scale.

Excellent: 1
Very good: 2
Good: 3
Fair: 4
Poor: 5
This is a five point Likert scale

Numerical data

Numerical data on the other hand represent numbers. There are two types of numerical data namely: discrete and continuous.

Discrete data can usually be counted. For example, the number of learners in class, number of schools in a community, etc.

Continuous data is infinite and impossible sometimes to count. For example the height of a person can be 1.652m. Other examples of a continuous data are mass, area, distance, time, etc. Continuous data can take decimal numerals.

ACTIVITY

In groups, give examples of
(a) categorical data;
(b) numerical data.

BEST BRAIN

B8.4.1.1.2 Select and justify method to collect data

In Basic 6, we learnt how to prepare a questionnaire to collect data from people. Data collection can be done in different ways such as through:
Online
Interviews
Survey
Phone calls
Mail
Face-to-face, etc. depending on the type of data needed.

For example, if one wants to know how a teacher delivers a lesson on mathematics, we can prepare a four points Likert scale and ask each learner in the class to select one.

Example 741

Afful, a Basic 8 learner, is investigating how effective a teacher in Basic 7 of his school teaches mathematics. He prepared a five point Likert scale, administered to each learner in Basic 7 to answer using the following scale

	Excellent: 1	Very good: 2	Good: 3	Fair: 4	Poor: 5
Tick					

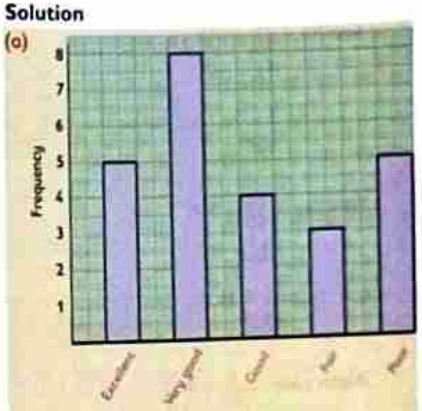
The result obtained is as shown.

Likert number	1	2	3	4	5
Frequency	5	8	4	3	5

(a) Draw a bar graph to represent the information.
(b) Find the percentage of learners who graded the teacher's delivery as poor.

Solution

(a)



(b) Total learners who answered the question
$= 5 + 8 + 4 + 3 + 5$
$= 25$
Percentage of learners who graded him
poor $= \frac{5}{25} \times 100\%$
$= 5 \times 4\%$
$= 20\%$

Example 742

The data shows the type of soap brand preferred by 50 learners in a class.

Type of soap	Geisha	Sunlight	Alata soap	Lux	Others
Number of learners	15	10	5	5	15

(a) Represent the information on a pie chart.
(b) What fraction of learners prefer sunlight?

BEST BRAIN

Solution

(a) Total number of learners: $50 = 360°$

Geisha $= \frac{15}{50} \times 360°$
$= \frac{15 \times 36°}{5}$
$= 3 \times 36°$
$= 108°$

Sunlight $= \frac{10}{50} \times 360°$
$= \frac{10 \times 36°}{5}$
$= 2 \times 36°$
$= 72°$

Alata soap $= \frac{5}{50} \times 360°$
$= \frac{5 \times 36°}{5}$
$= 1 \times 36°$
$= 36°$

Lux $= \frac{5}{50} \times 360°$
$= \frac{5 \times 36°}{5}$
$= 1 \times 36°$
$= 36°$

Others $= \frac{15}{50} \times 360°$
$= \frac{15 \times 36°}{5}$
$= 3 \times 36°$
$= 108°$

(b) Fraction of learners who prefer sunlight
$\frac{15}{50} = \frac{1}{5}$

EXERCISE 210

Date.....

1. The table shows the result obtained by a survey conducted by Kumi on educational level of 30 parents.

Educational level	Primary	JHS	SHS	Degree	Masters
Frequency	8	7	6	5	4

(a) Draw a bar graph to represent the information.
(b) What educational level are most parents found?

2. A shop owner designed a customer experience sheet on how customers who come to his shop feel: Very poor:1, Poor:2, Neutral:3, Good:4, Very good:5. The answers provided by 40 customers are as shown.

Likert number	1	2	3	4	5
Frequency	6	6	10	8	10

BEST BRAIN

(a) Draw a bar graph to represent the information.
(b) Find the fraction of customers who feel the services rendered by the shop is neutral.

3. A Basic 8 learner conducted a survey on why youth would like to travel outside Ghana. The result is as shown in the table.

Reason to travel	Work	Study	Leisure	Health care	Others
Frequency	40	15	10	5	30

(a) Represent the information on a pie chart.
(b) State 2 other reasons why most youth want to travel.

4. The results in the table give the answers posed by Gabriella to her father on how he spends his monthly salary.

Item	Food	Transport	School fees	Utilities	Others
Amount (GH₵)	1,500	800	1,000	500	2,200

(a) Find the amount the father receives as salary each month.
(b) Represent the information on a pie chart.
(c) What percentage of his salary does he spend on others?

BEST BRAIN

B8.4.1.1.3 Organise data

Most data (primary) collected by a researcher (the one looking for the data) is ungrouped. An ungrouped data is the data you first collect from respondents (people who answer your questions) in a survey.

In this section, we shall learn how to put ungrouped data into groups according to given classes and represent them in graphical form.

Grouping data in classes

To put an ungrouped data into classes, we first count all the numbers in the first class, write its tally and the corresponding frequency. This is repeated for all other classes.

Example 743

The data shows the height of learners in Basic 8 as measured by their class teacher.

183 179 172 183 177 173 177 171
184 176 182 172 171 183 179 172
184 185 176 182 177 183 171 182
176 171 177 178 175 181

Prepare a frequency distribution table (height, tally, frequency) using the classes 171 – 173, 174 – 176, 177 – 179, 180 – 182, 183 – 185.

Solution

Height	Tally	Frequency
171 – 173	𝍸 ///	8
174 – 176	////	4
177 – 179	𝍸 //	7
180 – 182	////	4
183 – 185	𝍸 //	7

Example 744

The data shows the marks scored in a test by learners in Basic 8 at Asuansi Catholic School.

11 13 8 7 25 18 22 23 20
15 19 5 4 16 14 19 21 6
3 12 9 10 13 18 14

(a) Prepare a frequency distribution table (marks, tally, frequency) using the classes 1 – 5, 6 – 10, etc.

(b) How many learners are in the class?

Solution

Marks	Tally	Frequency
1 – 5	///	3
6 – 10	𝍸	5
11 – 15	𝍸 //	7
16 – 20	𝍸 /	6
21 – 25	////	4

(b) Number of learners in the class
= 3 + 5 + 7 + 6 + 4
= 25

EXERCISE 211 Date.....

1. The data shows the number of children in certain families.

3 5 7 4 10 3 4 3 1 2 1
9 10 3 4 2 3 1 5 6

Prepare a frequency distribution table (number of children, tally, frequency) using the classes 1 – 2, 3 – 4, 5 – 6, 7 – 8, 9 – 10.

BEST BRAIN

2. The data shows the number of acres of farms belonging to 30 farmers in a community.

7 10 15 1 3 10 14 20 17
9 11 18 12 2 11 16 18 3
20 5 19 14 20 3 8 7 13
18 12 1

Prepare a frequency distribution table (number of acres, tally, frequency) using the classes 1 – 5, 6 – 10, 11 – 15, 16 – 20, etc.

3. The data shows the marks scored in an exam by 30 learners in Basic 8 of Domiabra D/A Basic School.

51 98 64 44 61 100 65 88
95 87 74 56 62 50 43 93
71 66 70 82 69 97 84 49
54 64 90 78 79 77

Prepare a frequency distribution table (marks, tally, frequency) using the classes 41 – 50, 51 – 60, 61 – 70, 71 – 80, etc.

Representing organized data on waffle diagram, stem-and-leaf plot and box and whisker plot

In the previous section, we learnt how to organise data according to given classes. In this section, we will represent the organised data graphically.

Waffle diagram

A waffle diagram is a diagram that represents percentage of different categories. To draw a waffle diagram, we usually use a 10 by 10 cell grid with each cell representing a percentage point.

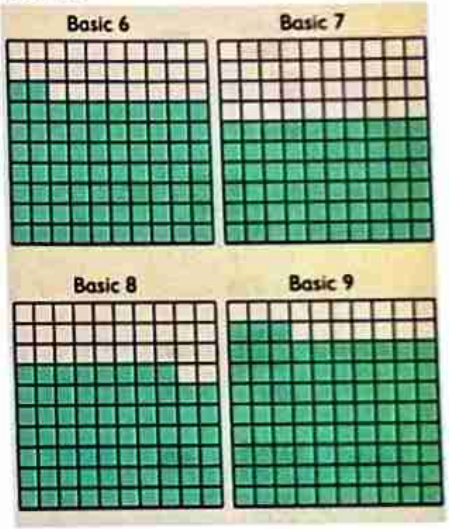
Example 745

The table shows the average score of Mathematics test by Basic 6 to 9 learners of Aboasi M/A.

Class	B6	B7	B8	B9
Average score (%)	72	60	68	83

Draw a waffle diagram to represent the information.

Solution



Example 746

The table shows total population of 3 districts in the Western Region of Ghana and those who have registered for health insurance scheme.

BEST BRAIN

District	Shama	Wassa East	Juaboso	Jomoro
Total population	150,000	160,000	100,000	125,000
Those on the scheme	90,000	120,000	90,000	100,000

Draw a waffle diagram to represent the information.

Solution

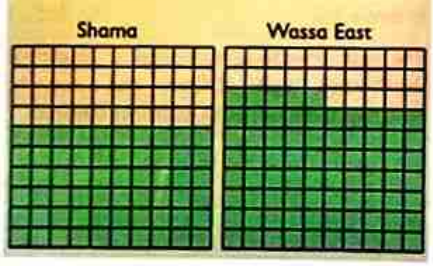
We first need to calculate the percentage of people who are on the scheme from the total population in each district.

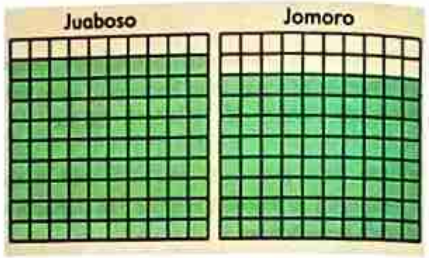
$$\text{Shama} = \frac{90000}{150000} \times 100\% = \frac{900}{15} = 60\%$$

$$\text{Wassa East} = \frac{120000}{160000} \times 100\% = \frac{1200}{16} = 75\%$$

$$\text{Juaboso} = \frac{90000}{100000} \times 100\% = 60\%$$

$$\text{Jomoro} = \frac{100000}{125000} \times 100\% = \frac{10000}{125} = 90\%$$





EXERCISE 212 Date.....

1. The table shows the profit made by a company over the past 3 years.

Years	Year 1	Year 2	Year 3
Profit (%)	53	48	69

Draw a waffle diagram to represent the information.

2. The table shows the total population of learners from B1 to B4 and those who are girls.

Class	B1	B2	B3	B4
Number of girls	21	20	12	27
Total population	35	50	40	45

Draw a waffle diagram to represent the information.

3. The table shows the percentage of those who contribute to SSNIT in 3 communities.

BEST BRAIN

Communities	Ekumfi	Saltpond	Moore
Adult population	24,000	32,000	25,000
SSNIT contributors	6,000	24,000	5,000

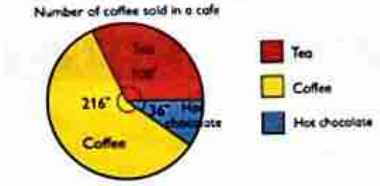
Draw a waffle diagram to represent the information.

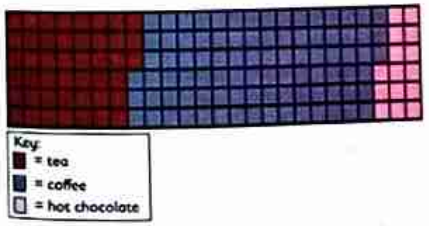
Interpreting Waffle Charts

Knowing how to draw waffle charts is not enough. You should also be able to interpret a given waffle chart. Interpreting a waffle chart can be likened to a pie chart.
Look at the charts below. The same information has been presented in a waffle chart and a pie chart.

Example 747

The waffle/pie chart shows the number of hot drinks sold in a cafe on a day. What is the percentage of tea?





Solution

From the waffle chart, there is a total of 150 drinks sold for all the three flavours.
Also, 45 cups of tea was sold.

$$\frac{45}{150} \times 100 = 30\%$$

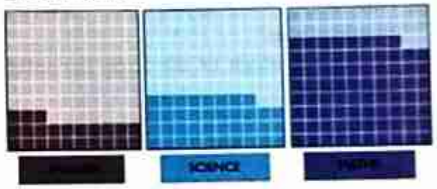
Therefore, 30% of tea was sold.

The same result will be provided by the pie chart
From the pie chart, the section representing tea is 108°.

$$\frac{108}{360} \times 100 = 30\%.$$

Example 748

The Waffle chart below shows the percentage score for English, Science and Mathematics of Methodist JHS in the 2023 BECE. Each square is a 10 x 10 grid.



a) What is the percentage score for each of the subjects?
b) What is the percentage difference between Maths and English?

Solution

a) Since each chart is a 10 x 10 grid, it means each small square is 1%.
Therefore,
English = 23%
Science = 38%
Maths = 78%

b) The percentage difference between Maths and English = 78 – 23 = 55%

BEST BRAIN

Example 749

The waffle chart below represents the number of different fruits havested in a secondary school farm. Each square represents 10 of a particular fruit.

a) How many more apples were harvested than mangoes?

b) What percentage of the fruits harvested were oranges?

Solution

Key ☐ = 10

a) From the chart there are 38 of the squares representing apples. Therefore, there were 38 x 10 = 380 apples harvested.
Also, there were 26 x 10 = 260 mangoes harvested.
The difference is 380 – 260 = 120.
Therefore, there were 120 more apples than mangoes.

b) The number of oranges harvested is 36 x 10 = 360
Total number of fruits is 360 + 260 + 380 = 1000.

Percentage for oranges is $\frac{360}{1000} \times 100 = 36\%$

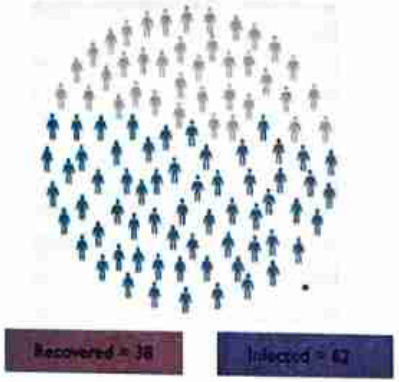
Example 750

The Waffle chart below is an info graphic of the number of COVID-19 infected persons and number of recoveries in the Central Region of Ghana.

a) If it requires GH¢800.00 to treat a patient, how much money has been spent on the recovered individuals?

b) Each infected person requires 2 nose masks per day. What percentage of nose masks is required if there are only 100 nose masks left on a particular day?

c) If a patient is selected at random, what is the probability that he/she is an infected person?



Solution

a) There are 38 recovered individuals
38 x 800 = GH¢30,400
Therefore, GH¢30,400 has been spent on recovered patients

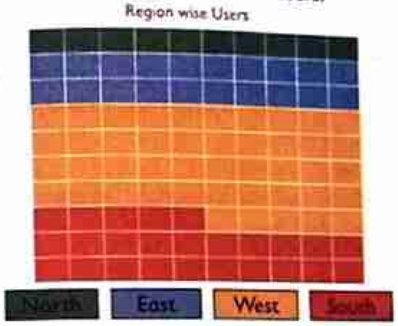
b) There are 62 infected persons
If each infected person requires 2 nose masks per day, then for that day the treatment facility will require 124 nose masks.
If there are 100 available, then they will need 24 more (i.e. 124 – 100 = 24)
This translate into $\frac{24}{124} \times 100 = 19.4\%$

c) Total number of infected and recovered patients = 100
Number of infected persons = 62
P(selecting an infected person) = $\frac{62}{100} = \frac{31}{50}$

BEST BRAIN

EXERCISE 213 Date

1. The Waffle chart below shows prevalence of mobile phone use in some zones in Ghana. Use the chart to answer the questions that follow.
Note: A square represents 100 users.

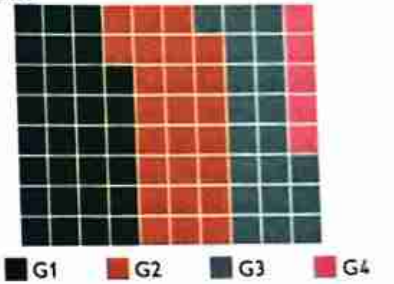


a) How many users were recorded for East?
b) What is the percentage of users representing the South?
c) How many more users are in the South than East?

2. The Waffle chart shows the favourite colours of some JHS2 students. Use the chart to the answer questions that follow.

a) How many students were surveyed for the research?
b) What percentage of learners preferred the deep green colour?
c) What is the probability of selecting a student who prefers a red colour?

3. The waffle chart represents the number of votes cast for four band groups at a competition. Each square represents 50 votes.



a) What is the difference in votes between G1 and G3?
b) What is the total number of votes cast?
c) What is the percentage of votes cast for G4?

4. The Waffle Chart below shows the top 100 city destinations in the world. The cities have been identified under some continents.

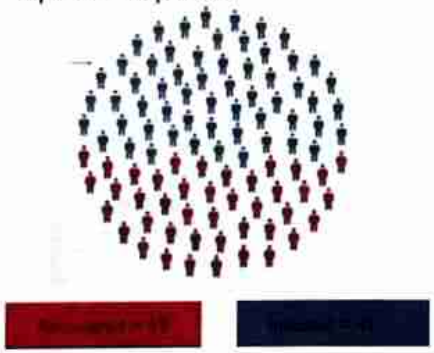
each square represents one city, unranked

BEST BRAIN

- Asia Pacific
- Australasia
- Eastern Europe
- Latin America
- Middle East and Africa
- North America
- Western Europe

a) How many countries are in Asia Pacific according to the chart?
b) What percentage of the countries are in Latin America?
c) What is the fraction of the countries that are in Western Europe?

5. The waffle chart below is an info graphic of the number of COVID-19 infected persons and number of recoveries in the Greater Accra Region of Ghana. The chart shows 59 robots for recovered patients and 41 robots for the infected patients. Each robot represents 10 patients.



a) How many patients have recovered?
b) If 38% of the infected persons are males, how many females are infected?
c) If a patient is selected at random, what is the probability that he/she is an infected person?

Stem-and-leaf plot

In this section, we shall learn how to represent an ungrouped data into a stem-and-leaf plot.
A stem-and-leaf plot is a special table where each data value is split into two parts namely: the stem (the first digit or digits) and a leaf (the last digit).
To represent an ungrouped data into a stem-and-leaf plot, the following steps are considered.

- Determine the smallest and largest number in the data.
- Identify the stems.
- Draw a vertical line and list the stems to the left of the line.
- Complete the leaves.

Example 751

The data below represent the marks scored by 20 workers in a promotion examination.

21 17 29 5 7 25 28 29 19 15 9
24 26 19 20 20 28 14 8 26

Draw a stem-and-leaf plot for the data.

Solution

Step 1: Determine the smallest and the greatest number.
The smallest number is 5 and the greatest number is 29.
Step 2: Identify the stem.
Stem for the smallest number is 0 and the stem for the greatest number is 2.
Step 3: Draw a vertical line and list the stem.

Stem
0
1
2

Step 4: Write the unit digit to the right of the line (the leaves).

BEST BRAIN

Stem	Leaf	Key
0	5 7 8 9	0 | 5 means 5
1	4 5 7 9 9	
2	0 0 1 4 5 6 6 8 8 9 9	

Example 752

The data shows the height (in cm) of learners of Basic 9 in St. Simons Basic School in Papagya.

160 156 142 135 122 124 133 144 148
152 161 153 155 123 141 131 147 150
155 126 143

(a) Draw a stem-and-leaf plot for the data.
(b) Two learners have the same height, what is the height?

Solution

(a)

Stem	Leaf
12	2 3 4 6
13	1 3 5
14	1 2 3 4 7 8
15	0 2 3 5 5 6
16	0 1

(b) The height is 155cm

Interpreting Stem and Leaf Plots

After learning to draw stem and leaf plots, you should also be able to interpret it and solve problems on them. Before we solve some examples on interpreting stem and leaf plots, let us take a look at some misconceptions.

Common misconceptions

There are some misconceptions that learners have about stem and leaf plots. These misconceptions lead to common errors that students make when interpreting stem and leaf plots. Let's discuss a few of them.

- Incorrectly reading the numbers in the diagram
  For example, for the following stem and leaf diagram, the value of 13 | 5 is misinterpreted as the value 5, instead of 135.
- Miscounting
  A common error is to incorrectly count the numbers in a stem and leaf diagram when calculating an average or range. It is a good idea to mark off each number to avoid missing a number out
- More than one digit for the leaf
  The leaf must only be one digit, otherwise, the data can be greatly misinterpreted. Make sure that the smallest place value for the data is used for the leaf.

Below is an example of a stem and leaf plot and its parts.
"32" is split into "3" (stem) and "2" (leaf).
More examples:

- Stem "1" Leaf "5" means 15
- Stem "1" Leaf "6" means 16
- Stem "2" Leaf "1" means 21
- Etc.

Example 753

The stem and leaf diagram below shows the ages of teachers in a particular school.

Ages of teachers

2	2	4	7	9		
3	1	2	5	5	7	9
4	0	4	4	7		
5	2	7	9			

Key: 36 years = 3|6

BEST BRAIN

a) Find the range of ages.
b) Find the modal age.
c) Find the median age.

Solution

a) Range = 59 – 22 = 37
b) Modal age = 35 and 44
c) Median age:
There are 17 numbers
So, (17 + 1) ÷ 2 = 9th value
Therefore, median age = 37.

Example 754

The stem and leaf diagram below shows the weight of 10 puppies in kilograms.

Weight of 10 puppies

1	9
2	2 8
3	1 4 4 9
4	5 8
5	1

Key: 3 | 9 represents 3.9 kg

Find:
a) mode
b) median
c) range
d) mean

Solution

a) The mode is the most frequent value, so the mode for this set of data is 3.4 kg, as it occurs twice.
b) The median is the middle value. As there are 10 values, the median is located in the position $\frac{10+1}{2} = 5.5$.
Count 5 and a half places, in this case that is between 3.4 and 3.4 (add them up and divide by 2) gives us a median of 3.4 kg.
c) The range is calculated by subtracting the lowest value from the highest value, giving us 5.1 – 1.9 = 3.2 kg.
d) The mean is calculated by adding up all the numbers and dividing by the number of values so, 1.9+2.2+.....+5.1 = 35.1.
Dividing by 10, we get a mean of 3.51 kg.

EXERCISE 214 Date.....

1. The data shows the number of story books read by some learners in a year:
7,20,8,4,15,17,22,11,8,7
(a) Draw a stem-and-leaf plot for the data.
(b) Find the median number of books read by the learners.

2. The data shows the ages of some workers in a factory
26 29 39 42 32 27 44 48 37 20
30 31 43 30 49 33 27 39 41 28
Draw a stem-and-leaf plot for the data.

3. The data below shows the number of times 13 learners in Mr Appiadu's class walked to school in two weeks.
7 8 3 13 14 25 5 17 20 18 6
22 15
(a) Draw a stem-and-leaf plot for the data.
(b) Find the median number of times the learners walked to school.

4. The data below shows the examination score of Mills from Basic 1 to Basic 5.
69 81 75 98 71 87 92 90 92 68
88 92 86 97 73
(a) Draw a stem-and-leaf plot for the data.
(b) Find the median score.
(c) Mills has the same score in three of the terms, what is the score?

5. Adam measured the height of all the people in his choir. The values, in cm, are given below.
154,180,176,153,154,180,176,153,162,
165,154,186,162,165,154,186,190,187,
176,176,190,187,176,176,172,182,177,
169, 172,182,177,169,
Draw a stem-and-leaf diagram to represent Adam's data.

BEST BRAIN

Interpreting dual stem and leaf diagram

Comparing data sets is simplified by using a dual stem and leaf diagrams which have two sets of data represented back to back.
For example, here are two sets of data showing test scores of 20 males and 20 females.

	Female
0	6 7
1	2 4 8 8 9
2	4 5 5 5 6 6
3	0 1 2 3 3 3 6
4	

Key: 1 | 2 means 12

	Male
0	1 4 5
1	0 2 3 4 4 8
2	2 7 7 9
3	0 0 0 3 6 8
4	0

Key: 1 | 0 means 10

By combining them together to form one dual stem and leaf diagram, we can directly compare the two sets of data.

Female		Male
7 6	0	1 4 5
9 8 8 4 2	1	0 2 3 4 4 8
6 6 5 5 5 4	2	2 7 7 9
5 3 3 3 2 1 0	3	0 0 0 3 6 8
	4	0

Key: 4 | 1 | 3 represents
14 Female
13 Male

Note: Please be careful not to interpret 14 females as 41. This is because in a dual stem and leaf diagram, the structure might make you see the numbers in the stem at the back of the leaves on the left and this might lead to wrong interpretations.

Example 755

This dual stem and leaf diagram shows the results for the students in Miss Mensah's class.

Boys		Girls
	0	5 6 9
2	1	0 4
9 4 1	2	6
8 8 5 3 0	3	2 3 3
5 1	4	0 7

Boys Key: 1 | 4 represents 41 marks
Girls Key: 4 | 0 represents 40 marks

a) Find the range of the boys' results.
b) Find the median of the girls' results.
c) Find the modal mark for the boys' results.
d) Compare the boys' and girls' results.

Solution

a) The range is found by subtracting the lowest from the highest values on the 'boys' side of the diagram is 45 – 12 = 33.
The range is 33.

b) The median is the middle value on the girls' side of the diagram. There are 11 values so the middle value is found by adding 1 and dividing by 2. (11 + 1) ÷ 2 = 6th place.
The median is therefore 26.

c) The mode is the most common number.
Here, the mode is 38 as it is the most frequent value for the boys.

d) We need to compare the median and range for each set of data:
The boys did better on average (median = 33) than the girls (median = 26).
The girls' marks (range = 42) were more spread out than the boys (range = 33).

BEST BRAIN

EXERCISE 215 Date.....

1. The stem and leaf diagram below shows heights of Mrs Aidoo's flowers in centimetres.

Key: 1 | 2 means 12	
0	9
1	2 8
2	1 3 3 4 9
3	5 8
4	1

   a) What is the range of the height of Mrs Aidoo's flowers?
   b) What is the modal height of Mrs Aidoo's flowers?

2. The stem and leaf diagram below shows the ages of a group of people at a party.

Key: 3 | 5 represents 35 years	
3	5 7 9
4	1 4 5 8
5	0 3 6

   a) How many people are there in the group?
   b) What age is the youngest member of the group?
   c) What age is the oldest member of the group?
   d) How many people are under 45?
   e) How many people are 45 and over?

3. Below is a stem and leaf diagram showing the scores of 18 javelin throws in a school sports day. The longest throw wins.

Key: 1 | 3 means 13 metres	
0	5 8 9
1	0 0 4 6 7
2	0 1 2 5 6 9
3	0 1 2 4

   What was the mean distance thrown?

4. The stem and leaf diagram below represents a sample of the lengths of leaves from a tree.

Key: 6 | 4 represents 6.4 cm	
5	4 4 4 5 6 8
6	0 1 2 4 6 7 7 8
7	0 1 3 4
8	
9	8

   Which average would be the most suitable to represent the data?

5. The stem and leaf diagram below shows the maximum weight of a deadlift lifted by each competitor in a weightlifting competition. Everyone managed to record at least one lift.

Key: 22 | 5 means 225 lbs	
21	0 0 5 5
22	0 0 0 0 0 5 5 5 5
23	0 0 0 5

   What was the median weight lifted?

BEST BRAIN

6. The head of school is looking at the Basic 9 mock exam results for Maths and Science The results are shown below in the stem and leaf diagram:

Maths		Science
	3	2 6 9
9 7 6 2	4	1 8
9 3 1	5	6 7 7 9
6 5 5 2	6	0 2 5 7 8
7 2 1 0 0	7	3 5 9
7 1 0	8	8 9

   Key: 2\|4\|1 means
   42 Maths
   41 Science

   a) What is the modal score for Science?
   b) What is the modal score for Maths?
   c) What is the difference between the range for Maths and the range for Science?
   d) Compare the median value for the two subjects and comment on the learners' performance.

7. 42 people competed in a local 3 legged race. The time for each pair was recorded in a back to back stem and leaf diagram.

Adults		Children
	13	1 3 4 8
	14	2 2 4 5 8
	15	1 5 9
	16	
	17	1 4
	18	

   Key: 3\|15\|2 represents
   15.3 seconds for Adults
   15.2 seconds for Children

   Select the completed stem and leaf diagram for the adult race using the results below.

   16.3s 18.1s 15.9s 16.1s
   17.4s 18.9s 17.5s

Adults		Children
9	13	1 3 4 8
3 1	14	2 2 4 5 8
4 5	15	1 5 9
1 9	16	
	17	1 4
	18	

Adults		Children
	13	1 3 4 8
	14	2 2 4 5 8
9	15	1 5 9
1 3	16	
4 5	17	1 4
1 9	18	

Adults		Children
	13	1 3 4 8
	14	2 2 4 5 8
	15	1 5 9 9
	16	1 3
	17	1 4 4 5
	18	1 9

Adults		Children
	13	1 3 4 8
	14	2 2 4 5 8
9	15	1 5 9
3 1	16	
5 4	17	1 4
9 1	18	

BEST BRAIN

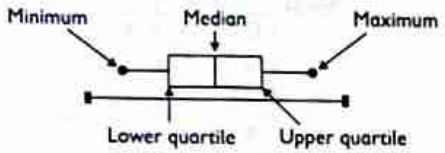
Box and whisker plot

A box and whisker plot is a diagram that shows the distribution of a set of data usually presented on a number line The plot displays a 5 number summary on it namely: the minimum, the maximum, the first quartile, median and the upper quartile.

To draw a box and whisker plot, the following steps are considered.

- Arrange the data set from least to greatest.
- Identify the highest and the lowest values in the data set.
- Find the median or the middle number in the data set.
- Find the lower quartile (median of the numbers less than the median mark).
- Find the upper quartile (median of the numbers more than the median mark).

In a box plot, we draw a box from the first quartile to the third quartile and a vertical line goes through the box at the median as shown.



Example 756

The data shows the age of 10 women at an antenatal department of Nsorkor hospital on a particular day.

35 29 30 35 38 23 26 36 34 27

(a) Draw a box and whisker plot for the data.

(b) Find the probability of selecting a woman from the group who is above 30 years.

Solution

(a) Order the data:

23, 26, 27, 29, 30, 34, 35, 35, 36, 38

$\text{Median} = \frac{30+34}{2} = \frac{64}{2} = 32$

First quartile: median of the data points less than the median.

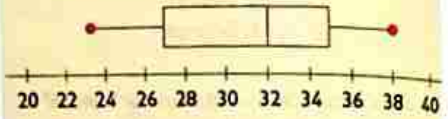
23, 26, 27, 29, 30

First quartile = 27

Upper quartile: median of the data points more than the median.

34, 35, 35, 36, 38

Upper quartile = 35



(b) Those above 30 years = 5

$\text{Probability} = \frac{5}{10} = \frac{1}{2}$

Example 757

The data shows the marks scored by 15 learners in Berekum R/C Basic School in Mathematics test.

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

(a) Draw a box and whisker plot for the data.

(b) What fraction of the learners scored not less than 26?

Solution

(a) The data set are already ordered.

$\text{Median position} = \frac{n+1}{2} = \frac{15+1}{2} = \frac{16}{2}$

$= 8$ th

Median = 27

BEST BRAIN

First quartile: median of the data points less than the median.

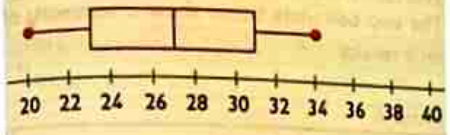
20, 21, 22, 23, 24, 25, 26

First quartile = 23

Upper quartile: median of the data points more than the median.

28, 29, 30, 31, 32, 33, 34

Upper quartile = 31



(b) Those who scored not less than 26 (those who scored 26 or more) = 9

$\text{Fraction} = \frac{9}{15} = \frac{3}{5}$

EXERCISE 216 Date.....

1. The data shows the ages (in years) of 8 siblings in a house.
   2 5 7 8 10 12 13 15
   (a) Draw a box and whisker plot for the data.
   (b) Find the probability of selecting a child whose age is even.

2. The data shows the pressure of 10 women at a hospital.
   125 126 128 130 132 133 137 138 140 142
   (a) Draw a box and whisker plot for the data.
   (b) What fraction of the women have their pressure less than 130?

3. The data shows the marks obtained by 13 applicants at an interview.
   60 61 62 63 64 65 66 67 68 69 70 71 72
   (a) Draw a box and whisker plot for the data.
   (b) Find the probability of those who passed the interview, if the pass mark was 70.

4. The data shows the temperature of 10 children in a hospital.
   32 33 35 36 37 38 39 40 41 42
   (a) Draw a box and whisker plot for the data.
   (b) What percentage of the children have normal temperature? If normal temperature is from 36 to 38.

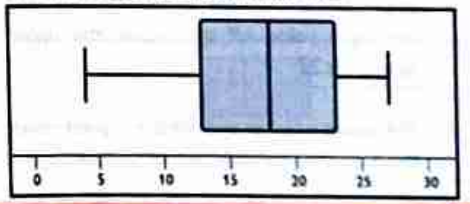
5. The data shows the number of bags of maize a farmer harvested in 5 years from both the main and lean season.
   12 13 14 15 17 20 23 24 25 28
   Draw a box and whisker plot for the data.

Interpreting box and whisker plots

Learning how to draw a box and whisker plot for a given data set is not enough. You also need to be able to interpret a given box plot. Let's take a look at some examples.

Example 758

The box plot below displays a given data set. Identify the lower quartile, upper quartile, minimum value and maximum value.



BEST BRAIN

Solution

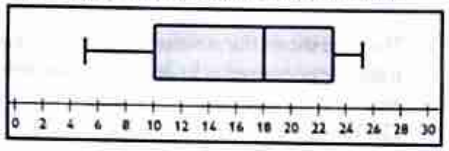
The box runs from the lower quartile 13 to the upper quartile 23.
The whiskers join to the box;
The lower whisker ends at the minimum value of 4 and the upper whisker ends at the maximum value of 27.

Example 759

This table shows some descriptive statistics for a set of data.

Minimum Value	5
Lower Quartile	
Median	16
Upper Quartile	
Maximum Value	

This box plot represents the same set of data



a) Identify the lower quartile.
b) Identify the upper quartile.
c) Identify the highest value.

Solution

a) The lower quartile is the vertical line on the box plot at the lower end (left end) of the box. Reading the data value off the x-axis, the lower quartile is 10.

b) The upper quartile is the vertical line at the upper end (right end) of the box. Reading the data value off the x-axis, the upper quartile is 23.

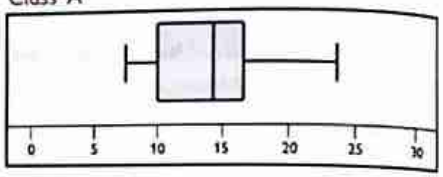
c) The upper whisker ends at the highest value; this is 25. The completed table is, therefore,

Minimum Value	5
Lower Quartile	10
Median	16
Upper Quartile	23
Maximum Value	25

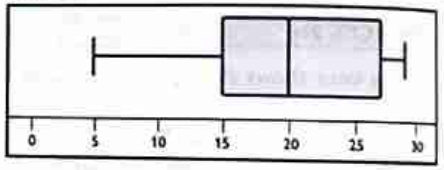
Example 760

Two classes of students sat the same Maths test. The two box plots below show a summary of their results.

Class A



Class B



Compare these two sets of data.

Solution

Compare the medians to comment on the average.
The median for Class A is 14.
The median for Class B is 20.
This means that 'on average, Class B scored better marks on the test than Class A'.
Alternatively, you could state something like 'the median mark in Class A was less than the median mark in Class B'.

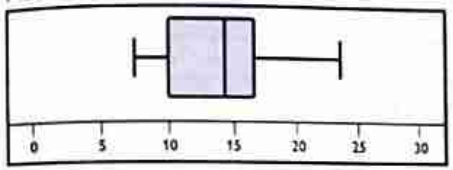
Example 761

Class A (see Example 760) also sat an English test. Their marks are summarised below. Compare the distributions of marks on the Maths and English tests.

English

Minimum Value	10
Lower Quartile	20
Median	24
Upper Quartile	27
Maximum Value	30

Maths



a) Convert the data into the same format.
b) Compare the medians to comment on the average.
c) Compare the IQR (or range) to comment on the spread or consistency.

Solution

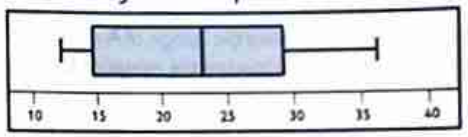
a) If we read the key data value from the Maths box plot, we get the following five-number summary for Maths marks:

Minimum Value	7
Lower Quartile	10
Median	14
Upper Quartile	16
Maximum Value	23

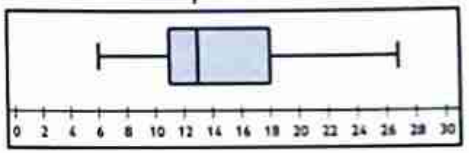
b) The median value for English is 24.
The median value for Maths is 14.
This means that 'on average, the class scored higher marks on the English test than on the Maths test'.

EXERCISE 217 Date.....

1. Construct a table to show this five-number summary on the box plot.

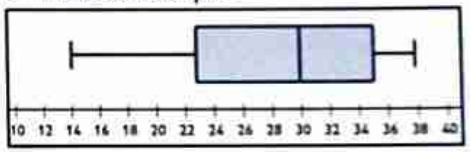


2. Look at this box plot.



What is the value of the lower quartile?

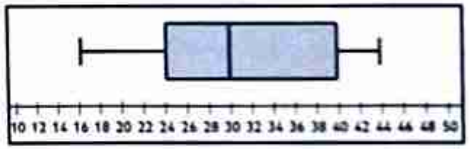
3. Look at this box plot.



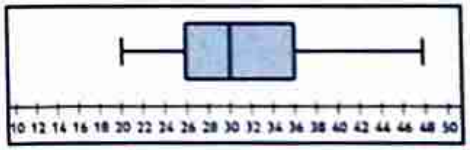
What is the value of the median?

4. Look at these two box plots and choose the incorrect statement:

A:



B:

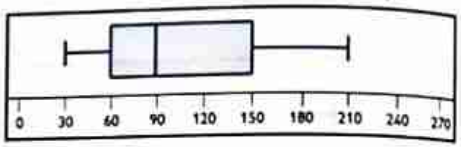


a) A and B have the same median.
b) A and B have the same range.
c) The maximum value of B is larger than the maximum value of A.
d) The interquartile range of A is smaller than the interquartile range of B.

5. Here is some information about the length of time people spent in a shopping centre on a weekend.

Shortest time	60 minutes
Longest time	4 hours
Lower quartile	1 hour 30min
Interquartile range	1 hour 30min
Median	2 hours 30mins

The box plot below shows the lengths of time people spent at a shopping centre on a weekday.



Make two comments to compare the distributions.

BEST BRAIN

BB.4.1.1.4 Calculate the mean, median and mode

In Basic 7, we learnt how to calculate mean and median of both grouped and ungrouped data. In this section, we shall use our knowledge from Basic 7 to calculate for mean, median and introduce the concept of mode.

Mode

Mode is the most frequent occurred number or item in a set of data.

Example 762

The data below shows the ages of 7 children in a house.

5 15 7 3 15 11 9

(a) Find the modal age.
(b) Find the probability of selecting a child who is more than 10 years.

Solution

(a) The most occurring age is 15
Therefore the modal age is 15
(b) Those more than 10 years are 3

$$\text{Probability} = \frac{3}{7}$$

Example 763

The data below shows the number of items bought from a shop in a day.

Item	Television	Refrigerator	Computer	Rice cooker
Frequency	4	3	10	7

Find the modal item.

Solution

The item with most occurring frequency is computer.

Therefore the modal item is computer.

Note: When a data set has two numbers that occur with the same frequency, the set is called bimodal. In that case we write both numbers.

Example 764

The data below shows the ages of some children at a birthday party.

4 3 5 7 6 5 4 2 8 5

(a) Find the modal age.
(b) What is the mean age of the children?
(c) Find the median age.
(d) Find the probability of selecting a child who is 5 years old.

Solution

Arranging the numbers in ascending order, we have

2, 3, 4, 4, 5, 5, 5, 6, 7, 8

(a) The modal age is 5

(b) $\text{Mean age} = \frac{2+3+4+4+5+5+5+6+7+8}{10}$

$= \frac{49}{10}$

$= 4.9$

(c) Total number of children is 10 (even)

$\frac{n}{2} = \frac{10}{2} = 5$

The number at the 5th position is 5

$\frac{n}{2} + 1 = 5 + 1 = 6$

The number at the 6th position is 5

$\text{Median} = \frac{5+5}{2} = 5$ years

(d) Those who are 5 years old = 3

$\text{Probability} = \frac{3}{10}$

BEST BRAIN

We can observe that the answers for mode, mean and median are all around the number 5. This number describes the center of the data distribution.

Measure of central tendency of a data set can be found by using mean, median and mode.

Central tendency is a measure that represents the single value of the entire data set. From the example above, we can say that the number 5 represents the center of the data set.

Example 765

The data below shows the mass of 20 fruits in a basket.

Mass (x)	1	2	3	4	5
Frequency (f)	3	$m+2$	m	3	2

(a) Calculate the value of m.
(b) Find the
  (i) mode;
  (ii) median;
  (iii) mean.
(c) Find the probability of picking a fruit that has a mass more than 3 kg.

Solution

(a) Since the total number of fruits is 20, we shall add all the frequencies and equate to 20.

$3+m+2+m+3+2=20$

$2m+10=20$

$2m=20-10$

$2m=10$

$\frac{2m}{2}=\frac{2m}{2}$

$m=5$

Mass (x)	1	2	3	4	5
Frequency (f)	3	7	5	3	2

(b) (i) The highest frequency is $5+2=7$, therefore mode is 2

(ii) Total frequency is 20 (even)

$\frac{n}{2}=\frac{20}{2}=10$

The number at the 10th position is 2

$\frac{n}{2}+1=10+1=11$

The number at the 11th position is 3

Median $=\frac{2+3}{2}=2.5$

(iii) Mean $=\frac{1\times3+2\times7+3\times5+4\times3+5\times2}{20}$

$=\frac{3+14+15+12+10}{20}$

$=\frac{54}{20}$

$=2.7$

(c) Fruits that have a mass of more than 3 kg $=3+2=5$

Probability $=\frac{5}{20}=\frac{1}{4}$

Example 766

The data below shows the number of hours Ampiah studied each day.

3 4 5 6 5 3 5 3 5 6 3 5 4 5 4

(a) Prepare a frequency distribution table for the data.
(b) Find the
  (i) mode;
  (ii) median;
  (iii) mean.

BEST BRAIN

Solution

(a)

Time	Tally	Frequency
3	////	4
4	///	3
5	//// /	6
6	//	2

(b) (i) The highest frequency is 6, therefore mode is 5

(ii) Total frequency is 15 (odd)

Median position $=\frac{n+1}{2}=\frac{15+1}{2}=\frac{16}{2}$

$=8$th

We shall add up frequencies starting from 4 until we get 8 or more $4+3+6=7+6=13$, therefore the median is 5

(iii) Mean $=\frac{3\times4+4\times3+5\times6+6\times2}{15}$

$=\frac{12+12+30+12}{15}$

$=\frac{66}{15}=4.4$

Example 767

The table shows the marks scored by some learners in a class test.

Marks	5	6	7	8	9	10
Number of learners	2	n	$n+1$	1	2	1

(a) If the mean of the data is 7, find the value of n.
(b) Find the (i) mode; (ii) median.
(c) Find the probability of selecting a learner who scored the modal mark.

Solution

(a) Mean $=\frac{5\times2+6\times n+7(n+1)+8\times1+9\times2+10\times1}{2+n+n+1+1+2+1}$

$7=\frac{10+6n+7n+7+8+18+10}{7+2n}$

$7=\frac{53+13n}{7+2n}$ (multiply both sides by $7+2n$)

$7(7+2n)=53+13n$

$49+14n=53+13n$

$14n=53+13n-49$

$14n=4+13n$

$14n-13n=4$

$n=4$

Marks	5	6	7	8	9	10
Number of learners	2	4	5	1	2	1

(b) (i) Mode is 7.

(ii) Total number of learners is 15 (odd)

Median position $=\frac{n+1}{2}=\frac{15+1}{2}$

$=\frac{16}{2}=8$th

Therefore median is 7.

(c) Those who had the modal mark = 5

Probability $=\frac{5}{15}=\frac{1}{3}$

BEST BRAIN

EXERCISE 218

Date.....

1. The data below shows the number of phones sold in the first 10 days.
   6 7 9 6 4 6 5 3 7 9
   Find the modal age.

2. The data below shows the weight (in kg) of pregnant women who attended a clinic on a particular day.
   68 61 64 68 70 65 64 72 70 64 67 64 63 62 69
   Find the modal weight.

3. The data below shows the number of hours 10 learners used to study at home on each Saturday.
   4 5 4 8 7 3 4 4 5 6
   (a) Find the (i) mode; (ii) median.
   (b) Calculate the mean time.
   (c) Find the probability of selecting a learner who studied for 5 hours.

4. The data below shows the ages of children at a Sunday school.
   8 7 6 9 8 5 8 9 5 5 8 8 5 7 7 6 8 9 5 8
   (a) Prepare a frequency distribution table for the data.
   (b) Find the (i) mode;
   (ii) median;
   (iii) mean.
   (c) Find the probability of selecting a child who is less than 7 years old.

5. The data below shows the marks scored by 15 people at an interview.

Marks (x)	6	7	8	9	10
Number of people	2	6	4	2	1

   Find the
   (a) mode;
   (b) median;
   (c) mean;
   (d) probability of selecting a person who had a modal mark.

BEST BRAIN

B8.4.1.1.5 Find appropriate measure of central tendency

In the previous section, we learnt how to find the mode, median and mean of a data set. We also learnt that mode, median and mean are ways of finding measure of central tendency. In this section, we shall use graphs to interpret which type of measure of central tendency is appropriate to be used from a given data.

Example 768

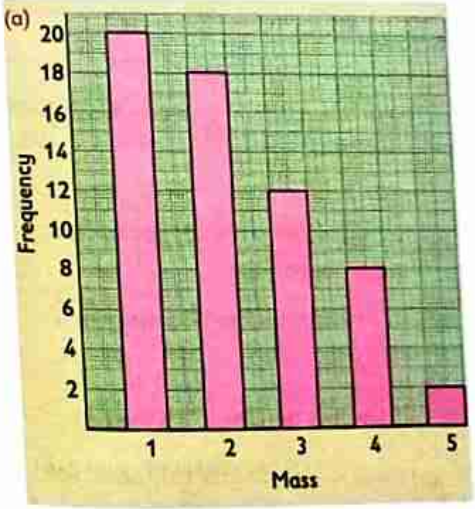
The table shows the mass (in kg) of 6 different fruits in a box.

Mass (kg)	1	2	3	4	5
Frequency (f)	20	18	12	8	2

(a) Draw a bar graph for the data.
(b) Find the (i) mode;
(ii) median;
(iii) mean.

Solution

(a)



(b) (i) Mode is 1 kg.
(ii) Total frequency is 60 (even)

$\frac{n}{2} = \frac{60}{2} = 30$

The number at the 30th position is 2

$\frac{n}{2} + 1 = 30 + 1 = 31$

The number at the 31st position is 2

Median $= \frac{2+2}{2} = 2$

Therefore the median is 2 kg.

(iii) Mean $= \frac{1\times20 + 2\times18 + 3\times12 + 4\times8 + 5\times2}{20 + 18 + 12 + 8 + 2}$

$= \frac{20 + 36 + 36 + 32 + 10}{60}$

$= \frac{134}{60}$

$= 2.23$ kg

Example769

The table shows the number of litres of water drunk by some patients at a hospital in a given day.

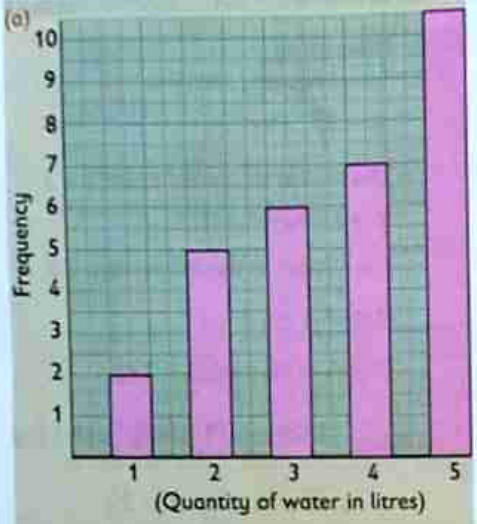
Quantity of water (in litres)	1	2	3	4	5
Frequency (f)	2	5	6	7	10

(a) Draw a bar graph for the data.
(b) Find the (i) mode;
(ii) median;
(iii) mean.
(c) Find the probability that a patient selected at random from the hospital drunk more than 4 litres of water.

BEST BRAIN

Solution

(a)



(b) (i) The mode is 5

(ii) Total frequency = 2 + 5 + 6 + 7 + 10 = 30 (even)

$\frac{n}{2} = \frac{30}{2} = 15$

The number at the 15th position is 4

$\frac{n}{2} + 1 = 15 + 1 = 16$

The number at the 16th position is 4

Median $= \frac{4+4}{2} = 4$

Therefore the median is 4 litres

(iii) Mean $= \frac{2\times1 + 5\times2 + 6\times3 + 7\times4 + 10\times5}{30}$

$= \frac{2 + 10 + 18 + 28 + 50}{30}$

$= \frac{108}{30}$

$= 3.6$

c) Those who drunk more than 4 litres are 10

Probability $= \frac{10}{30} = \frac{1}{3}$

We observe that the mode, median and mean are all different. We can also observe that the graph is shifted (skewed).

When we have a skewed distribution, the median is often the best measure of central tendency.

Example 770

The table shows the length (in metres) of logs a company has at the warehouse.

Length of (in metres	1	2	3	4	5	6
Frequency (f)	1	3	4	6	4	2

(a) Find the (i) mode;
(ii) median;
(iii) mean.

(b) What percentage of the logs have length between 2 m and 5 m?

Solution

(a) (i) The mode is 4.

(ii) Total frequency = 1 + 3 + 4 + 6 + 4 + 2 = 30 (even)

$\frac{n}{2} = \frac{20}{2} = 10$

The number at the 10th position is 4

$\frac{n}{2} + 1 = 10 + 1 = 11$

The number at the 11th position is 4

Median $= \frac{4+4}{2} = 4$

Therefore the median is 4 metres

(iii) Mean $= \frac{1\times1+3\times2+4\times3+6\times4+4\times5+2\times6}{20}$

BEST BRAIN

Mean $= \frac{1 + 6 + 12 + 24 + 20 + 12}{20}$

$= \frac{75}{20}$

$= 3.75$

(b) Number of logs with length between 2 m and 5 m are 10.

Probability $= \frac{10}{20} \times 100\%$

$= 50\%$

The graph in (a) shows some form of bell shape (symmetry).

From (b), we can observe that the mode, median and mean (to nearest whole number) are all the same.

We can always use mode, median or mean to measure the central tendency when we have symmetrical distribution for continuous data.

Example 771

The table shows the responses made by customers on services rendered by a shop.

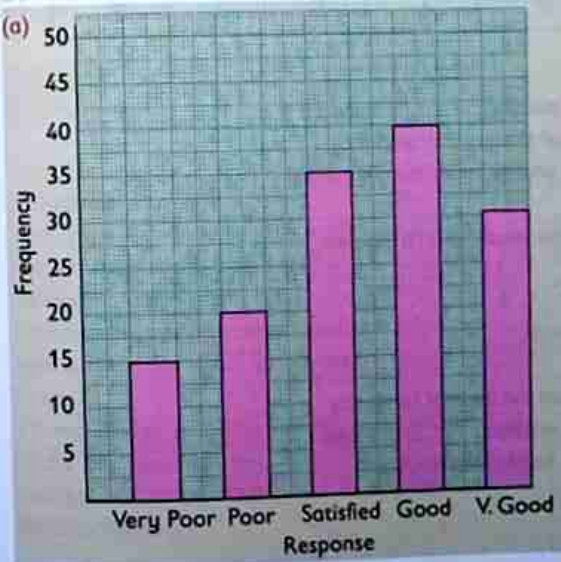
Response	Very Poor	Poor	Satisfied	Good	Very Good
Frequency	15	20	35	40	30

(a) Draw a bar graph for the data.

(b) Find the mode.

Solution

(a)



(b) The mode is the Good response.

We use the mode as a measure of central tendency for categorical data.

BEST BRAIN

EXERCISE 219 Date.....

1. The table shows the responses given by some learners in a class about the teaching methodology of their Mathematics teacher.

Response	Fair	Good	Very good	Excellent
Frequency	9	12	7	1

(a) Draw a bar graph for the data.
(b) State the appropriate measure to find the central tendency.

2. The table shows the ages (in years) of some learners in Basic 8 class.

Ages (in years)	11	12	13	14	15
Number of learners	2	3	5	6	7

(a) Draw a bar graph for the data.
(b) State the appropriate measure to find the central tendency.

3. The table shows the marks scored by the learners in Basic 8 of Akuse M/A Basic School.

Marks	5	6	7	8	9	10
Number of learners	7	7	6	5	3	2

(a) Draw a bar graph for the data.
(b) Find the (i) mode; (ii) median; (iii) mean.
(c) State the appropriate measure to find the central tendency.
(d) Find the probability of learners who scored more than 8 marks.

4. The table shows the height (in cm) of some seedlings in a garden.

2 4 3 3 4 4 3 4 3 3 4
2 5 3 5 6 5 5 4 2

(a) Prepare a frequency table for the data.
(b) Draw a bar graph for the data.
(c) State the appropriate measure to find the central tendency.
(d) Find the (i) mode; (ii) median; (iii) mean.
(e) What fraction of the seedlings have height less tan 4cm?

B8.4.1.2.1 Justify a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings.

Choosing the best average (mean, median and mode) to represent a given data set

Whenever we calculate the measures of central tendencies, we are to select the best among the three to represent our data. The nature of the data you collect will sometimes require the use of a particular measure of central tendency to represent that data. Let us take a look at some real-life problems and the best measure/average to represent that given data.

Example 772

Mrs. Salifu sells footwear at Techiman market. Every week she sells different sizes of footwear. The data below shows the different sizes of footwear sold in a particular month.

40 42 40 42 43 42 40 44 42 42 44
42 44 40 42 44 42 42 40 42 40 42
44 42 43 42 44

Which size should Mr Salifu purchase more for his shop?

Solution

This type of data is a nominal scale. The footwear sizes can be treated as symbols. It is interesting to note that different countries have different ways of indicating shoe sizes and so the type used in this data is only one of the many kinds.

Therefore, for this data, we will only find the mode to represent our data. The mean and median will automatically not be appropriate for this data set.

Frequency distribution table

Size of shoes (x)	Frequency (f)
40	6
42	13
43	2
44	6

Mode = size 42

This means that since size 42 is the size most purchased by customers, Mr Salifu should purchase more for his shop instead of the other sizes.

Example 773

A group of workers wanted to make contributions to buy items for a Friday night get together. The data below shows the suggestions of how much they wish each of them should pay.

¢7 ¢10 ¢15 ¢200 ¢12
¢8 ¢7 ¢8 ¢7 ¢10
¢12 ¢15 ¢10

a) Calculate the mode, median and mean.
b) State with reasons the average that is considered the best for this data.

Solution

Frequency distribution table

Amount (GH¢) (x)	Frequency (f)	fx
7	3	21
8	2	16
10	3	30
12	2	24
15	2	30
200	1	200

Mode = GH¢7 and GH¢10

Median = $\frac{\Sigma f}{2} = \frac{13+1}{2} = 7$

The median is the 7th number.

Therefore, from the table our median = GH¢10.

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{321}{13} = 24.70$

Mean = GH¢25.00 (to the nearest whole number).

Justification of the best average for the distribution

From the calculations, the best measure is the median.

This is because there were two different modal values. This makes it difficult to choose one.

Also, since there was an outlier or an extreme value, it affected the mean. As can be seen by the value of the mean which is too far away from the amount that most of the friends suggested. This will not be easily accepted by the majority of the friends.

Example 774

The frequency table below shows the frequency distribution of goals scored in each game by Accra Hearts of Oak FC. On the average, how many goals did the team score in that season? Calculate the mean, median and mode and select the measure that best represents the data.

Goals (x)	Frequency (f)	fx
0	4	0
1	1	1
2	4	8
3	1	3
4	2	8
5	3	15
6	2	12
7	1	7

Mode = 2.

Median = $\frac{\Sigma f}{2} = \frac{18}{2}$ 9th and 10th

$\therefore$ Median = $\frac{2+3}{2} = \frac{5}{2} = 2.5$ goals

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{54}{18} = 3 = 3$ goals

From the calculations, the best measure to represent this data is the mean. This is because all the goals were involved in the calculation of the mean. It means that, on average, the team scored three goals per match in the season.

EXERCISE 220 Date

1. A class teacher wanted to choose a score to represent the performance of his class in a mathematics test. Below is the distribution of the test. Determine the mode, mean, and the median for the distribution. Choose which of the measures best represents the class. Give reasons for your answer.

Marks (x)	Frequency (x)	fx
3	3	9
4	4	16
5	4	20
7	2	14
8	1	8
9	3	27
10	4	40

2. The following are marks obtained in a Creative Arts test for 30 learners. Organise the data into a frequency distribution table. Calculate the mean, median and mode and select the best measure for the distribution. Explain your reasoning.

BEST BRAIN

STRAND 4.2 CHANCE OR PROBABILITY

B8.4.2.1.1 Perform probability experiment involving two independent events

Edem

Hawa

We can observe that Edem has tossed a coin. The total outcome of tossing a coin is {Head, Tail}. It means that the coin can land on Head (H) or Tail (T).

Hawa is also about to roll a ludo die, the total outcome of rolling a die is {1, 2, 3, 4, 5, 6}. It means that when a die is rolled, any one of the numbers can appear. We can clearly see that the outcome of tossing a coin and the outcome of rolling a die does not have anything in common. Two or more events that do not have anything in common (the occurrence of one event does not affect the other) are called independent events.

Example 775

Gyan and Sylvia are two learners in the same school. Gyan goes to school by a bicycle whilst Sylvia goes to school by foot. Does the coming to school of Gyan depends on that of Sylvia?

Solution

The occurrence of one event does not affect the other. Therefore the events are independent.

Example 776

Ampomah's father is a casual laborer in a company who is paid daily. Ampomah can only go to school on a given day if the father went to work on the previous day. Is the occurrence dependent or independent?

Solution

The event is dependent because Ampomah can only go to school if the father goes to work.

BEST BRAIN

Example 777

A sack contains some fruits of the same size. There are 7 oranges, 5 mangoes and 8 apples.

(i) List the elements of the sample space.

(ii) Find the probability of selecting a mango from the sack.

(iii) Find the probability of picking an apple from the sack.

Solution

(i) Sample space = {7 oranges, 5 mangoes, 8 apples}

Total sample space = 7 + 5 + 8

= 20

(ii) There are 5 mangoes in the sack and each of them can be picked.

Probability of picking a mango = $\frac{5}{20}$

= $\frac{1}{4}$

(iii) Probability of picking an apple = $\frac{8}{20}$

= $\frac{2}{5}$

EXERCISE 221 Date.....

1. Abugri and Sasu attend the same school. Sasu always goes to school by Abugri's father's car. Is the event of Abugri and Sasu going to school dependent or independent?

2. Mrs. Gyan goes to farm on any given day provided it did not rain the previous night. Is the event of Mrs. Gyan going to farm dependent or independent?

3. Esi and Nii are two learners in the same school. Nii goes to school by foot and Esi goes to school by car. Does Esi going to school depend on that of Nii or not?

4. A wallet contains 5 ten cedis notes, 10 twenty cedi notes and 20 one cedis notes all of the same size.
   (i) List the elements of the sample space.
   (ii) Find the probability of picking a 5 cedi note.
   (iii) Find the probability of picking a twenty cedi note.

5. A drawer contains some coins of the same size. Three of them are 50 pesewa coin, five of them are 1 cedi coins and the rest are 2 cedis coins. If the probability of selecting a 2 cedis coin is $\frac{1}{5}$. Find the number of 2 cedis coins in the drawer.

BEST BRAIN

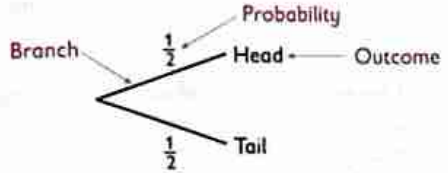
B8.4.2.1.2 Find the probability of an event using tree diagram or tables

In the previous section, we explained the difference between events that are dependent and those that are independent. In this section, we shall learn how to find the probability of an event(s) which involves selecting an item more than ones. To explain this concept, we shall use a tree diagram or tables.

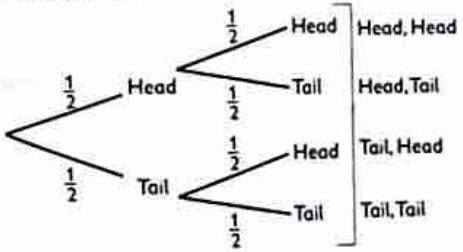
Tree diagram

Tree diagram is a diagram that uses branches to show combination of two or more events. Each branch is labelled at the end with its outcome and the probability written on it.

For example, we know that when a coin is tossed, it has two outcomes {H, T}. We can use a tree diagram to show the probability of each outcome as



We can extend the tree diagram to when a coin is tossed 2 times.



Finding the probability of the first toss being head and the second toss being head is $\frac{1}{2} \times \frac{1}{2}$ as shown on the diagram.

Example 778

A six sided fair die is tossed 2 times, find the probability of

(a) obtaining a six in the first toss and not a six in the second toss;

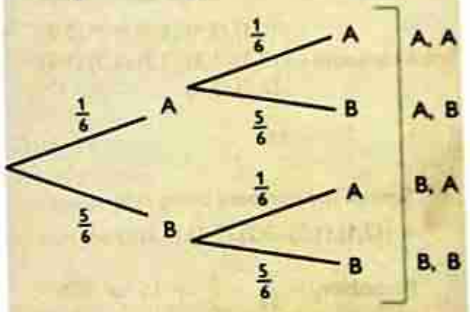
(b) obtaining not a six in the first toss and not a six in the second toss; using a tree diagram.

Solution

Total outcome = {1, 2, 3, 4, 5, 6}

Let A = a six and B = not a six

$P(A) = \frac{1}{6}$, $P(B) = \frac{5}{6}$



(a) $P(A) = \frac{1}{6} \times \frac{5}{6}$

$= \frac{5}{36}$

(b) $P(B) = \frac{5}{6} \times \frac{5}{6}$

$= \frac{25}{36}$

BEST BRAIN

Example 779

Fiifi has three oranges labeled 1, 2, 3 and four mangoes labeled 1, 2, 3 and 4, all in different baskets. He picks one fruit from each basket, find the probability that the sum
(a) of the numbers on the fruits is odd;
(b) is a multiple of 3; using a table.

Solution

	Basket 1			
		1	2	3
Basket 2	1	(1, 1)	(2, 1)	(3, 1)
	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)
	4	(1, 4)	(2, 4)	(3, 4)

Total outcome = {(1,1),(2,1),(3,1),(1,2),(2,2),(3,2),(1,3),(2,3),(3,3),(1,4),(2,4),(3,4)}

= 12

(a) Sum of the numbers being odd
= {(2,1),(1,2),(3,2),(2,3),(1,4),(3,4)}

Probability = $\frac{6}{12} = \frac{1}{2}$ or 0.5 or 50%

(b) Sum of the numbers being a multiple of
= 3{(2,1),(1,2),(3,3),(2,4)}
= 4

Probability = $\frac{4}{12} = \frac{1}{3}$ or 0.333 or $33\frac{1}{3}\%$

EXERCISE 222 Date.....

1. A fair coin is tossed 2 times.
   (a) Write down all the outcomes
   (b) Find the probability of obtaining
      (i) 2 heads;
      (ii) only one tail.

2. A fair coin is tossed 3 times.
   (a) Write down all the outcomes.
   (b) Find the probability of obtaining
      (i) only 2 heads;
      (ii) all tails.

3. A jar contains 5 black and 3 red marbles. A marble is selected from the jar, the colour is observed and put back into the jar and the second marble is selected. What is the probability that
   (a) the two marbles selected are all black;
   (b) one of the marbles is black, use tree diagram.

4. A fair 6-sided die is rolled two times, find the probability of obtaining the sum being more than 10 using a table.

5. Two sets P and Q are defined by $P = \{1,2,3,4\}$ and $Q = \{5,6,7,8\}$.
   A number is selected from each of the sets.
   (a) Construct a table to show all the possible outcomes.
   (b) Calculate the probability that the sum is
      (i) an even number,
      (ii) a prime number.

B8.4.2.1.3 Perform a probability experiment involving two independent events such as drawing coloured bottle tops from a bag with replacement and listing the elements of the sample space.

Example 780
A bag contains 3 red balls and 4 black balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is
(i) black
(ii) not black.

Solution:
(i) Total number of possible outcomes
= 3 + 4 = 7.
The number of favourable outcomes for the event E.
= Number of black balls = 4

So, P (E) = $\frac{\text{Number of Favourable Outcomes for the Event E}}{\text{Total Number of Possible Outcomes}}$
= $\frac{4}{7}$

(ii) The event of the ball being not black = $\bar{E}$
Hence, required probability = $P(\bar{E})$
= 1 - P(E)
= $1 - \frac{4}{7}$
= $\frac{3}{7}$

Example 781
A lot of 24 bulbs contain 25% defective bulbs. A bulb is drawn at random from the lot. It is found to be not defective and it is not put back. Now, one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Solution:
25% of 24 = $\frac{25}{100} \times 24 = 6$

So, there are 6 defective bulbs and 18 bulbs are not defective.

After the first draw, the lot is left with 6 defective bulbs and 17 non-defective bulbs.
So, when the second bulb is drawn, the total number of possible outcomes = 23 (= 6+ 17).
The number of favourable outcomes for the event E = number of non-defective bulbs = 17.
So, the required probability = P(E) = $\frac{17}{23}$

Example 782
A bag contains playing objects in different shapes. There are 4 blue triangles, 5 green squares and 7 red circles. A single object is drawn at random from the bag. What is the probability that the object is;
(i) not a circle
(ii) a triangle
(iii) not a triangle
(iv) not a square
(v) a circle
(vi) a square

Solution:
Number of blue triangles in a container = 4
Number of green squares = 5
Number of red circles = 7
Total number of objects = 4 + 5 + 7 = 16

i) The object is not a circle
P(the object is a circle)
= $\frac{\text{Number of circles}}{\text{Total number of objects}}$
= $\frac{7}{16}$
P(the object is not a circle)
= 1 – P(the object is a circle)
= $1 - \frac{7}{16} = \frac{16-7}{16} = \frac{9}{16}$

ii) The object is a triangle:

P(the object is a triangle)

$= \frac{\text{Number of triangles}}{\text{Total number of objects}}$

$= \frac{4}{16}$

iii) The object is not a triangle:

P(the object is a triangle)

$= \frac{\text{Number of triangles}}{\text{Total number of objects}}$

$= \frac{4}{16}$

P(the object is not a triangle)

= 1 – P(the object is a triangle)

$= 1 - \frac{4}{16} = \frac{16-4}{16} = \frac{12}{16} = \frac{3}{4}$

iv) The object is not a square:

P(the object is a square)

$= \frac{\text{Number of squares}}{\text{Total number of objects}}$

$= \frac{5}{16}$

P(the object is not a square)

= 1 – P(the object is a square)

$= 1 - \frac{5}{16} = \frac{16-5}{16} = \frac{11}{16}$

v) The object is a circle:

P(the object is a circle)

$= \frac{\text{Number of circles}}{\text{Total number of objects}}$

$= \frac{7}{16}$

vi) The object is a square:

P(the object is a square)

$= \frac{\text{Number of squares}}{\text{Total number of objects}}$

$= \frac{5}{16}$